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Concept lattice formalisms of Hébert's "semic analysis" and "analysis by classification"

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Abstract: In this article we provide a mathematical frame to the generation of class taxonomies suggested by Hébert in his analysis of the poem <<*Quelle affaire!*>> ('A Sorry Business!') by Gilles Vigneault (b. 1928) as well as a formalization of the structure of semic isotopies in his reading of *The golden ship* by Émile Nelligan (1879–1941). We also examine the characteristics of inter- and intra-semic molecules at work within Réne Magritte's painting *La clef des songes*. Our mathematical frame is Ganter and Wille's extension of lattice theory called *formal concept analysis*, for which we explore various formalisms and constructs that allow us to reason on semic structures.

Keywords: François Rastier; Louis Hébert; semic analysis; analysis by classification; formal concept analysis

1 Introduction

In Louis Hébert's (2020) recent book *An Introduction to Applied Semiotics*, the author explores structural semiotics from the perspective of the Paris school, in particular, the groundwork laid by François Rastier (1997, 2016) and A. J. Greimas (1983, 1987). Hébert provides well-considered introductions to a number of analytic methods, two of which we will focus on in the article – namely, *semic analysis* and *analysis through classification*. Both of these analytic methods are applied in numerous textual contexts by Hébert, though both methods emerge from the subtle distinctions and indirect definitions he gives in regard to *classes*, *types*, *tokens* and *elements*, in addition to how Rastier and Greimas conceived of the relation between a *seme*, *isotopy* and *molecule*. We begin by providing an overview of these terms and their usage.

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1.1 Nomenclature

1.1.1 Semes

Greimas and Courtés (1979: 278) define a seme as the "'minimal unit' of meaning" that is located on the "content plane" and corresponds to the *pheme*, the unit on the "expression plane." Like the constitution of phonemes as comprised of phemes, so too, semes (which we denote /seme/) are the components of *sememes*. But a seme itself is not autonomous or "atomistic," instead it "exists only because of the differential gap that opposes it to other semes" (Greimas and Courtés 1979: 278). This theoretical position is as much indebted to Saussure (1983), as to Greimas' recognition of how Levi-Strauss uses *constitutive units* for identifying signifieds in myths in the following form:

$$\frac{A}{\text{non} - A} \cong \frac{B}{\text{non} - B}. \tag{1}$$

Here, Greimas mirrors the notion of Levi-Strauss (1955) that "greater constitutive units" can be framed in terms of "distinctive features." That is to say that semes themselves follow this type of structure of opposition and form what Greimas calls *semic categories* (Greimas 1987: 16). These semic categories constitute the content plane, and therefore are considered anterior to the individual semes themselves. But there is also structure here too, which becomes fully formed and visualized through the semiotic square (Greimas and Rastier 1968: 88) via the utility of the relations of contradiction ($\leftarrow\cdots\rightarrow$), contrariety ($\leftrightarrow$) and implication ($\dashrightarrow$) between semes:

$$
\begin{tikzcd}
 & \mathbf{S} & \\
s_1 \arrow[rr, dotted, leftrightarrow] \arrow[drr, leftrightarrow] & & s_2 \arrow[dll, leftrightarrow] \\
\bar{s}_2 \arrow[u, dashed] \arrow[rr, dotted, leftrightarrow] & & \bar{s}_1 \arrow[u, dashed] \\
 & \bar{\mathbf{S}} &
\end{tikzcd} \tag{2}
$$

What is fundamental then to the semiotic square is *homologation*, or what Greimas and Courtés (1979: 144) consider "a rigorous formulation of reasoning by analogy" that they denote as A: B: A': B', which is a generalization of the case of the semiotic square as: $s_1 : s_2 :: \bar{s}_1 : \bar{s}_2$.

In terms of classifying semic categories themselves, Greimas and Courtés offer the following:

> An examination of different semic categories allows us to distinguish several classes: (a) **figurative semes** (or exteroceptive semes) are entities on the content plane of natural languages, corresponding to elements of the expression plane of the semiotics of the natural world, i.e., corresponding to the articulations of the sensory classes, to the perceptible qualities of the world; (b) **abstract semes** (or interoceptive semes) are content entities that refer to no exteriority, but which, on the contrary, are used to categorize the world and to give it meaning: for example, the categories relation/term, object/process; (c) **thymic semes** (or proprioceptive semes) connote semic microsystems according to the category euphoria/dysphoria, thus setting them up as axiological systems. (Greimas and Courtés 1979: 279)

Hébert (2020: 145–146) also specifies that semes can be thought of as *generic*, for which the corresponding sememe belongs to a *semantic class* as a semantic paradigm, made up of sememes. There are three types of generic semes, those that are *microgeneric*, *mesogeneric* and *macrogeneric*, for which all three correlate to the three semantic classes that Rastier (1997) contextualizes within the domain of his *interpretive semantics*. These semantic classes are *taxemes*, or the minimal classes by which sememes are distinguished, *domains*, which are connected to social contexts and human endeavors (e.g., dictionaries, scientific disciplines, etc.), and *dimensions*, or those most general classes of oppositions (e.g., animate vs. inanimate). Hébert give the example of the taxeme //tableware// and its three sememes, each of which

> contains the microgeneric seme /tableware/ and is distinguished from the other sememes of the same taxeme by a specific seme: /for piercing/ in 'fork,' /for cutting/ in 'knife' and /for containing/ in 'spoon.' Since this taxeme comes under the domain //food//, the three sememes also contain the mesogeneric seme /food/. (Hébert 2020: 146)

Unlike a generic seme, the *specific semes* highlighted by Hébert (2020: 145) are ones that distinguish a sememe from all other sememes of the same class. Hébert also distinguishes between *inherent semes* and *afferent semes*. In the case of the former, an inherent seme is one that belongs to a sememe's type and is *actualized* by default unless it is part of a *virtualized* structure. For example, 'albino crow' contains the inherent seme /black/ in the type of the sememe 'crow' that is virtualized because of 'albino.' Thus the seme /white/ here is actualized. Hébert notes that afferent semes are "present only in the sememe's token, that is, only by contextual indication" (Hébert 2020: 146). Thus if a seme is present in a context it is normally associated to, it is actualized, and if it is missing it is virtualized.

1.1.2 Isotopies and molecules

Greimas (1983: 81) describes an *isotopy* (denoted /isotopy/) as emerging from a *syntagm*, that is, a grouping of at least two semic figures. As Hébert (2020: 147) suggests, the sentence “I only use a knife for picking up peas” contains the (mesogeneric) isotopy /food/, which indexes the sememes ‘knife’ and ‘peas.’ In addition, it virtualizes the inherent specific seme /for cutting/ in ‘knife’ and actualizes the afferent seme /for picking up/. An isotopy can thus be extended over “two morphemes, two words, a paragraph, or a whole text” (Rastier 2016: 497). Thus given a pair of *generic isotopies*, that is, ones that correspond to one of the three semantic classes of taxeme, domain and dimension, we have what Rastier (1997: 35) identifies a *semic molecule*, or “the recurrence of groupings of relatively stable specific semes.” In Figure 1 we give Rastier’s diagram of the relation between these generic isotopies, and how their constituent sememes are covered by a semic molecule.

1.1.3 Classes, types, tokens and elements

In Ronald Schleifer’s introduction to Greimas’ *Structural Semantics*, he describes the assumption that for any process for which meaning is the translation of a sign into another system of signs, there must be present a corresponding *system* of elements. It should thus be possible to order these elements into *classes* (which we denote //class//) according to their possibilities of combination in addition to constructing a “general and exhaustive calculus of the possible combinations.” (Greimas 1983: xvi)

Hébert too describes a class as “not an individual entity, but rather an inventory of one or more properties, optionally accompanied by rules for evaluating the membership of the element.” (Hébert 2020: 11) For example, the semantic class //time of the day// contains the signifieds ‘day’ and ‘night.’ Regarding the evaluation of

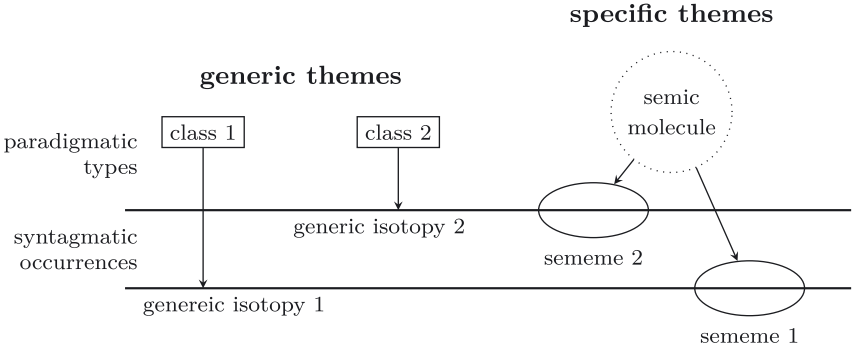


Figure 1: Rastier’s (1997: 36) diagram of the relation between themes, generic isotopies and a semic molecule.

membership to a semantic class, there is a similar notion in mathematical set theories such as NBG (von Neumann-Bernays-Gödel set theory) and ZFC (Zermelo-Fraenkel-Choice set theory) of the application of an *axiom of choice* on non-empty sets that allows for the construction of equivalence classes to describe elements from the union of these non-empty sets. In its most general form this is expressed as a *selector function* on a collection X of nonempty sets, such that for every set $A \in X$, $f(A)$ is an element of A:

$$\forall X \left[\varnothing \notin X \rightarrow \exists f : X \rightarrow \bigcup X \ \ \forall A \in X \ (f(A) \in A) \right]. \tag{3}$$

Such a selector function is what Hébert utilizes to distinguish between a genre of poetry (as a type) and a collection of say 'nature' poems (as a class):

> Strictly speaking, a type is not a class because rather than containing, or bringing together the token units (the poems) governed by it, it generates them. We will distinguish between a class's **extension** (or enumeration) and its **intension** (or comprehension). (Hébert 2020: 11)

An *element* is a member of a particular class found in its extension and satisfies an equivalence relation that is congruent to a selector function. Hébert considers a *token* as "more or less [a] complete manifestation of a model or type, such as a particular sonnet that is more or less regular" (Hébert 2020: 11). This causes some confusion as to the difference between a token and an element, as the token seems to not operate within the same set-theoretic universe of discourse as a class and its choice function. Hébert draws attention to this fact when he describes *typing* as an act of *categorization* in which "a token is subsumed under a type, related to it, and recognized as its emanation or manifestation" (Hébert 2020: 11). This neatly mirrors the difference between set theories and type theories in mathematics, where a type theory is merely a set of rules of inference to describe any *term* as a type and allow for operations and expected variable values while a set theory has both rules and *axioms*, or logical statements that are accepted to be true.

1.2 Motivation

Our motivation for this article is what we perceive as parallelisms between the analytic methods and conceptual frameworks of structural semiotics as espoused by Greimas (1983), Rastier (1997) and Hébert (2007, 2020), to the mathematics of extended lattice theory and "formal concept analysis" forwarded by Ganter and Wille (1999). In order to ground the connection between these fields, we examine the work of Hébert using his framework of "semic analysis" and "analysis by classification," which directly emerges from Rastier's interpretive semantics and Greimas' notion of semic categories.

For Wille (2005: 9), the mathematization of real-world concepts to discover underlying structures and their accompanying semantics through lattice constructions is a promising inter-disciplinary field because it has the potential to "activate implicit conceptions and experiences concerning the underlying domains." The mathematization of concepts thus points towards *subjective theories* which "contain implicit and explicit assumptions about objects and events, their conditions and causes, their characteristics, relations and functions" (Wille 2005: 8). This approach neatly mirrors what Rastier (1997: 5) asserts about objectivity in semiotic analysis, in which "meaning has no existence of its own beyond its utterance and its interpretation" Marrone (2022: 22). similarly notes that semiotic analysis ultimately seeks "to reconstruct the hierarchy of underlying layers" in subjective experiences. In our estimation, this is facilitated directly through imposed selector functions that seek to articulate, collate and render meaning in texts or images via associations between elements of a set.

To these ends, we firstly provide a mathematical frame to the generation of class taxonomies suggested by Hébert (2007: 167–169) in his analysis of the poem <<*Quelle affaire!*>> ("A Sorry Business!") by Gilles Vigneault. We also provide a formalization of the structure of semic isotopies through his reading of *The golden ship* by Émile Nelligan (Hébert 2020: 157–160) as well as the characteristics of inter- and intra-semic molecules at work within Réne Magritte's painting *La clef des songes* (Hébert 2020: 160–168). Through these examples, we introduce relative mathematical formalisms in order to provide a systematic description of the approach of structural semiotics to the conceptualization of a seme as the minimal unit of meaning, and how semes form the basis for larger emergent forms.

2 Analysis by classification

For Hébert, the cognitive act of *analysis by classification* is one that operates in a similar manner to the selector function we outlined in Equation (3). That is to say that the classifying agent chooses which feature(s) the elements must have in order to be part of the class, the values these features must take, and the rules for evaluating and determining membership (Hébert 2020: 205). In his *Dispositifs pour l'analyse des textes et des images* (Hébert 2007), an earlier edition to *An Introduction to Applied Semiotics*, he provides a sample analysis of the poem <<*Quelle affaire!*>> ("A Sorry Business!") (Vigneault 1998) (Figure 2) by the Quebec poet Gilles Vigneault (b. 1928). The poem reflects on a highly charged environmental and political affair during the 1980s in which the actress Brigitte Bardot visited the Canadian ice floes with a camera crew to denounce the hunting of baby seals and the seal fur trade. The successful campaign's ramifications saw new policies that caused a sharp decrease in the

The sand lance told the trout
The trout talked to the salmon
The salmon told the tuna
And so on down the line
From spring to lake and stream
It all goes to the river
All is turning from sand to stone
For all the people of the water
From the River to the sea
The bullhead and the catfish
Had just barely found out
When the sole and the herring
Were telling the cod about it
The capelin and the smelt
The turbot, the tilefish, the eel
Alerted their families
"Evacuate the Grand Banks!"

A lady of stupidity
In love with baby seals
Has taken her muff
For a walk on the ice floe . . .
And our common enemy
Any species of seal
Who kills and dismembers us
In December just as in June
The seal is multiplying
And no longer fears the hunter
Let us gently pack up and go
Said the sole to the flounder . . .
And that is how it came about
That there is nothing left to catch in the Great River
From Natashquan to Newf'ndland

Figure 2: The poem <<*Quelle affaire!*>> ("A Sorry Business!") by Gilles Vigneault (1998: 147–148), translated by Hébert (2007: 167).

harvesting of baby seals that caused an overall increase in the seal population which in turn affected fish numbers, and thus the livelihood of local fishing communities.

Hébert proposes a tree diagram (Figure 3) based in essence on what Greimas (1983: 123) identifies as the "simple opposition of immanence and manifestation." The diagram presents a differentiation between a class (unshaded node) as an *intension*, and an element (shaded node) as its *extension*. The tree is intended to also allow for the identification of *monadic* and *polyadic* classification structures. Hébert thinks of these as the difference between an element possessing a single class (monadic), or adhering to many classes (dyadic, triadic, *n*-adic) (Hébert 2007: 166).

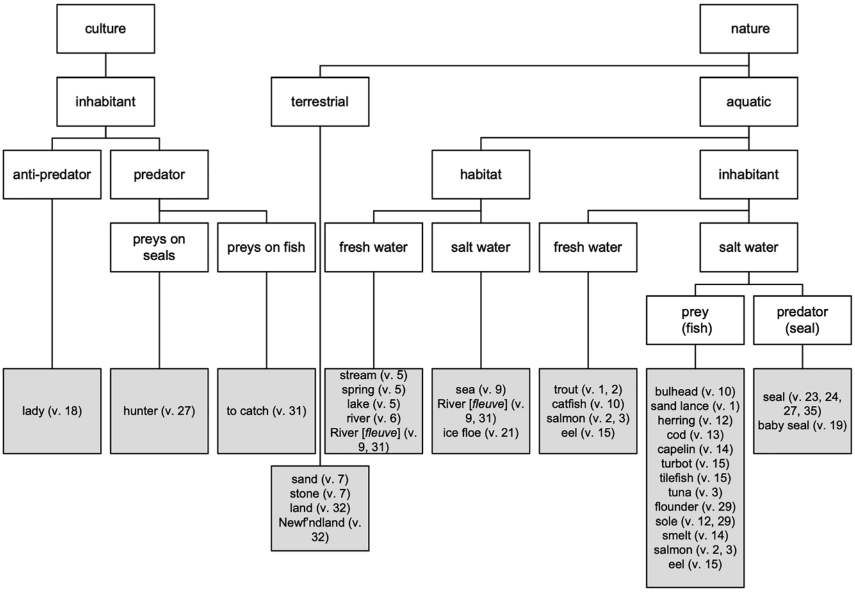
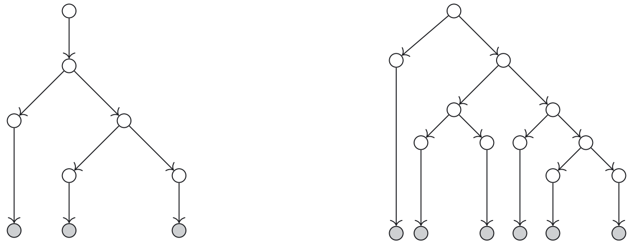


Figure 3: Hébert's (2007: 168) two disconnected finite rooted trees as a visualization of "Thematized classification in << *Quelle affaire!*>>."

But we can also think of Hébert's *monadic* and *polyadic* structures firstly within the realm of mathematical graph theory, and of finite directed graphs. This re-contextualization allows us to utilize the notion of a *path*. That is, given a finite directed graph (G, R), we call a path a finite sequence $a_0, \ldots a_n (n \geq 1)$ of elements (vertices) in G for which $(a_i, a_{i+1}) \in R$ for all i, such that $0 \leq i \leq n$. This allows us to subsequently introduce the notion of a *finite rooted tree* (Makinson 2020: 226).

Definition 2.1 (Finite rooted tree): We define a finite rooted tree as the directed graph $T = (G, R)$, where G is a set of vertices and R is a two place relation (a_i, a_{i+1}) over G. We call the root of G an element a, in which $\forall x \in G$ such that $a \neq x$, there is a unique path from a to x, but no path from a to itself.

Hébert's *monadic* and *polyadic* structures are thus equivalent to the *link-height* in a finite rooted tree. That is to say that when we set the root to level 0, a monodic classification is a child of link-height 1, and a polyadic classification is a chid of link-height that is greater than 1. Consider the following directed graphs A (left) and B (right) that represent unlabelled finite rooted trees from Hébert's example in Figure 3:



(4)

Here, Hébert's tokens or elements are the colored vertices or *leaves* of the finite rooted trees, such that for both A and B we have link-heights of greater than 1 from leaf to root: that is, a polyadic structure. But in terms of what Hébert calls *monoclassification* and *polyclassification*, he argues that

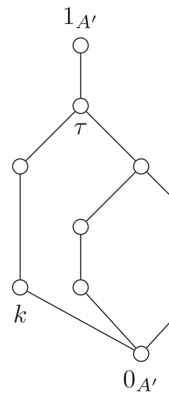
> one can distinguish between a 'vertical' polyclassification, which includes one or more encompassing classes (wolf < canines (subclass) < mammal (class)), and a 'horizontal' polyclassification, which is made at the same level of generality (human > werewolf < canines). The object being analyzed and/or the type of classification used in the act and/or the analysis of this act may allow for only single classifications of a single unit, or may admit multiple classifications. (Hébert 2007: 166)

Here, we can understand the distinction Hébert makes between a monoclassification and a polyclassification as the difference between a finite rooted tree and a *complete lattice*.

Definition 2.2 (Complete Lattice): An ordered set $\mathcal{L} \coloneqq (L, \leq)$ is a *lattice*, if for any two elements x and y in L, the supremum (greatest element) $x \vee y$, and the infimum (least element) $x \wedge y$ always exist. $\mathcal{L}$ is called a complete *lattice*, if the supremum $\bigvee X$, and the infinum $\bigwedge X$ exist for any subset X of L. Every complete lattice $\mathcal{L}$ has a greatest element, $\bigvee L$, or the unit *element*, $\mathbf{1}_L$, and a least element $\mathbf{0}_L$, the zero element.

Remark 1: We see in (4) that a finite rooted tree is a structure in which each node of the tree has exactly one parent, and thus no two diverging paths ever meet – Hébert's monoclassification. In contrast, a complete lattice is a structure in which any two nodes can be traced to greatest element or least *element*, which equates to what Hébert calls a polyclassification and the notion of classes and subclasses. Consider the following complete lattice A':

for which we see that the least upper bound or join between the elements i and k is the element τ, while the greatest lower bound or meet is the zero element. In fact for any two elements in A' there always can be found a meet or join in the form of two paths converging at the infimum $(0_{A'})$ or supremum $(1_{A'})$.



(5)

In the following sections we will introduce the mathematics of lattice theory in order to move from finite rooted trees to ordered concept lattices. The advantage of this recontextualization is that we can succinctly express semiotic notions such as Hébert's monoclassification and polyclassification using binary tables and their accompanying concept lattices, concept/subconcept relations, preconcepts constructions as well as attribute dependencies and attribute implications.

2.1 Formal concept analysis

Formal concept analysis (FCA) is a branch of mathematics pioneered by Rudolph Wille (1982) as a means in which to extend lattice theory. As Ganter and Wille (1999) contend, the basic notions of FCA center on a *formal context* and its *formal concepts*. These notions were developed in order to develop a system for formalizing "knowledge discovery" within any domain (Stumme et al. 1998: 451). FCA is a direct mathematization of 'conventional' concept through lexical fields such that concepts themselves are considered

> *cognitive acts* and *knowledge units* potentially independent of language. Only if they are used to give meaning to linguistic expressions, they become so-called *word concepts* which are conventualized and incorporated. The meanings of words for an individuum presuppose conceptual knowledge of that individuum which turns linguistic expressions into signs for those concepts. (Wille 2005: 6)

Like Hébert's recognition of the importance of considering how an intension of a term determines its extension, FCA provides a formalized method in which to study what Hébert calls classes and sub-classes, though in FCA these are known as *superconcepts* and *subconcepts*. Indeed the superconcept/subconcept relation in FCA was primarily influenced by how Frege (1952) identified the dual relationship between intension and extension and how each generates the other such that “the extension is the class of things in the world for which the intension is a true description” (Wilson and Keil 2001: 177). Below we introduce FCA through a number of standard definitions (Ganter and Wille 1999).

Definition 2.3 (Formal Context): A formal *context*, denoted $\mathbb{K}$, is a triple, (G, M, I), and consists of two disjoint sets; G, a set of *objects*, and M, a set of attributes as well as an incidence relation, I. An object g that has an attribute m, is denoted as gIm or $(g, m) \in I$.

Definition 2.4 (Subcontext): If $\mathbb{K} \coloneqq (G, M, I)$ is a formal context, and if $H \subseteq G$ and $N \subseteq M$, then $\mathbb{K}_n \coloneqq (H, N, I, H \times N)$ is a subcontext of $\mathbb{K}$.

Definition 2.5 (Apposition): Two formal contexts, $\mathbb{K}_1 \coloneqq (G, M_1, I_1)$ and $\mathbb{K}_2 \coloneqq (G, M_2, I_2)$, contain the same set of objects, G, such that $M_1 \cap M_2 = \varnothing$. The apposition of $\mathbb{K}_1$ and $\mathbb{K}_2$ is denoted as $\mathbb{K}_\sigma$, such that:

$$\mathbb{K}_1 \mid \mathbb{K}_2 \coloneqq (G, \dot{M}_1 \cup \dot{M}_2, \dot{I}_1 \cup \dot{I}_2), \tag{6}$$

for which $\dot{M}_j \coloneqq \{j\} \times M_j$ and $\dot{I}_j \coloneqq \{((j, g), (j, m)) \mid (g, m) \in I_j\}$ for $j \in \{1, 2\}$.

Let us first consider the formal context $\mathbb{K}_\sigma \coloneqq \mathbb{K}_1 \mid \mathbb{K}_2$ given in Figure 4. Here we use Hébert’s *elements*, or leaves from his finite rooted trees of << *Quelle affaire!*>> in Figure 3 to construct G, a set of objects for which a “×” in the binary table of $\mathbb{K}_\sigma$ indicates a incidence relation to a set of attributes M that correspond to what Hébert calls a collection of classes. In order to formalize the incidence relation of gIm in $\mathbb{K}_\sigma$, we relabel the classes so that they reflect in a similar manner how Greimas and Courtés (1979: 29–30) discuss *classemes* as “those semes which are recurrent in the discourse and which guarantee its isotopy.”

We construct $\mathbb{K}_\sigma$ through two contexts $\mathbb{K}_1$ and $\mathbb{K}_2$ that share the object set G, though split Hébert’s classes into M_1 and M_2 such that $M_1 \cap M_2 = \varnothing$. This follows Hébert’s two original disjunct finite rooted trees as classifications of the oppositional classes: //culture// and //nature//. From $\mathbb{K}_\sigma$ and its subcontexts $\mathbb{K}_1$ and $\mathbb{K}_2$ we can see that subsets of objects may share attributes and thus be considered formalized structures that form “*cognitive acts* and *knowledge units* potentially independent of language” (Wille 2005: 6). These units are called *formal concepts* and given some units may be contained in other units, leads to the notion of a partial order on a context and a hierarchy of concepts.

	is an actant	is from culture	is a predator	is a person	preys on seals	preys on fish	is an anti-predator	is from nature	is an animal	is terrestrial	is aquatic	is a fresh water habitat	is a salt water habitat	lives in fresh water	lives in salt water	is prey	is a seal	is a fish
lady	×	×		×			×											
hunter	×	×	×	×	×													
fisherman	×	×	×	×		×												
sand	×							×		×								
stone	×							×		×								
land	×							×		×								
Newf'ndland	×							×		×								
stream	×							×			×	×						
spring	×							×			×	×						
lake	×							×			×	×						
river	×							×			×	×						
River (fleuve)	×							×			×	×	×					
sea	×							×			×		×					
ice floe	×							×			×		×					
trout	×							×	×		×			×				×
catfish	×							×	×		×			×				×
salmon	×							×	×		×			×		×		×
eel	×							×	×		×			×		×		
bullhead	×							×	×		×				×	×		×
sand lance	×							×	×		×				×	×		×
herring	×							×	×		×				×	×		×
cod	×							×	×		×				×	×		×
capelin	×							×	×		×				×	×		×
turbot	×							×	×		×				×	×		×
tilefish	×							×	×		×				×	×		×
tuna	×							×	×		×				×	×		×
flounder	×							×	×		×				×	×		×
sole	×							×	×		×				×	×		×
smelt	×							×	×		×				×	×		×
seal	×		×			×		×	×		×				×	×	×	
baby seal	×		×			×		×	×		×				×	×	×	
Natashquan	×	×								×								

Figure 4: The formal context $\mathbb{K}_\sigma$ as the apposition of formal contexts $\mathbb{K}_1$ and $\mathbb{K}_2$, whose objects and attributes are sourced from Hébert's (2007: 168) two disconnected finite rooted trees that visualize the "Thematized classification in << *Quelle affaire!*>>" (c.f. Figure 3). The dashed line indicates the articulation of M_1 and M_2 such that M_1\coloneq {is an actant, is from culture, is a predator, is a person, preys on seals, preys on fish, is an anti-predator}, and M_2\coloneq {is from nature, is an animal, is terrestrial, is aquatic, is a fresh water habitat, lives in fresh water, lives in salt water, is prey, is a seal, is a fish}.

Definition 2.6 (Formal Concept): A pair (A, B), with $A \subseteq G$ and $B \subseteq M$, is a formal concept of a formal context, $\mathbb{K}$. We call A the *extent*, and B the intent of the formal concept, such that $A^{\uparrow} = B$ and $B^{\downarrow} = A$. The set of all formal concepts of the formal context $\mathbb{K}$ is notated as $\mathfrak{B}(G, M, I)$, for which we define the incidence relation as $I = \bigcup\{A \times B \mid (A, B) \in \mathfrak{B}(G, M, I)\}$.

Remark 2: Given $A \subseteq G$, the derivation operator $\uparrow$ yields:

$$A^{\uparrow} := \{m \in M \mid \forall g \in A : gIm\}, \tag{7}$$

or the set of attributes common to the objects in A, while the operator $\downarrow$ yields:

$$B^{\downarrow} := \{g \in G \mid \forall m \in B : gIm\}, \tag{8}$$

which is the set of objects which have all attributes in B given $B \subseteq M$. Thus $(A^{\uparrow\downarrow}, A^{\uparrow})$ and $(B^{\downarrow}, B^{\downarrow\uparrow})$ are always concepts, and $A^{\uparrow\downarrow}$ is the smallest extent containing A. A context $\mathbb{K}$ thus always contains both a set of all *extents*, denoted $\mathfrak{U}(G, M, I)$ and set of all intents $\mathfrak{I}(G, M, I)$. The intersection of intents is always another intent such that given an index set T, for each $t \in T$ and $A_t \subseteq G$, then

$$\left(\bigcup_{t \in T} A_t\right)^{\uparrow} = \bigcap_{t \in T} A_t^{\uparrow}, \tag{9}$$

which is also true of attributes and the intersection of extents.

Definition 2.7 (Superconcept/Subconcept): Given a pair of formal concepts, (A_1, B_1) and (A_2, B_2), we say (A_1, B_1) is a subconcept of (A_2, B_2) if $A_1 \subseteq A_2$ or dually $(B_2 \subseteq B_1)$. This is formalized as the relation: $(A_1, B_1) \leq (A_2, B_2)$: $\Leftrightarrow A_1 \subseteq A_2 (\Leftrightarrow B_1 \supseteq B_2)$. The pair (A_2, B_2) is thus a superconcept of (A_1, B_1). The set of all formal concepts of $\mathbb{K}$, together with the order relation, $\leq$, is a concept lattice which is denoted as $\underline{\mathfrak{B}}(\mathbb{K})$.

From our formal context $\mathbb{K}_\sigma$, consider the following two concepts, $\mathfrak{c}_1$ and $\mathfrak{c}_2$ found in the set of all formal concepts $\mathfrak{B}(\mathbb{K}_1)$:

$$\mathfrak{c}_1 := (\{\text{baby seal, seal}\}, \{\text{preys on fish, is a predator, is an actant}\}), \tag{10}$$

$$\mathfrak{c}_2 := (\{\text{hunter, fisherman, baby seal, seal}\}, \{\text{is a predator, is an actant}\}). \tag{11}$$

We can see that given $\mathfrak{c}_1$ contains objects that are a subset of the objects of $\mathfrak{c}_2$, we say that $\mathfrak{c}_1 \leq \mathfrak{c}_2$, or that $\mathfrak{c}_1$ is a subconcept of $\mathfrak{c}_2$. This is more readily seen through a *Hasse diagram* found in Figure 5, and Figure 6. Here we can visualize the order ($\leq$) on $\mathfrak{B}(\mathbb{K}_n)$ that generates $\underline{\mathfrak{B}}(\mathbb{K}_n)$. We find that the edges of a diagram that connect objects (whose labels are shaded and sit below vertices) to other vertices, some of which contain attributes (whose labels sit above vertices). The concept $\mathfrak{c}_1$ has an intent (upwards connecting edges) that we can denote using the derivation operator as:

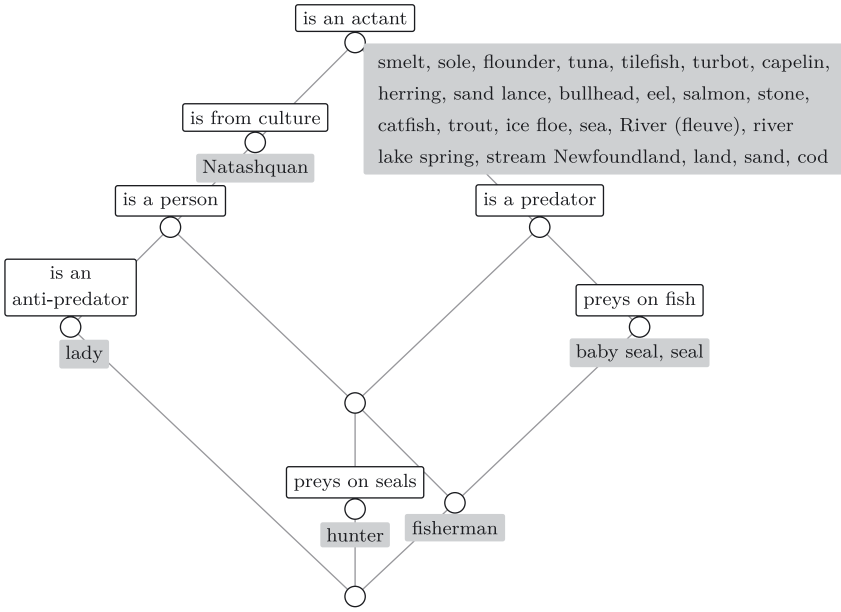


Figure 5: Hasse diagram of $\underline{\mathfrak{B}}\,(\mathbb{K}_1)$.

$$\{\text{baby seal, seal}\}^{\uparrow} \coloneqq \{\text{preys on fish, is a predator, is an actant}\}, \tag{12}$$

while for $\mathfrak{c}_2$ this is:

$$\{\text{hunter, fisherman, baby seal, seal}\}^{\uparrow} \coloneqq \{\text{is a predator, is an actant}\}, \tag{13}$$

given that the vertices where all objects of $\mathfrak{c}_2$ meet through upwards paths are the attribute vertices {is a predator} and {is an actant}. Indeed, given that $\underline{\mathfrak{B}}\,(\mathbb{K}_\sigma)$ is isomorphic to a complete lattice $\mathcal{L}$ (Definition 2.2) through Ganter and Wille's (1999: 20–21) proof of their *basic theorem on concept lattices* via the mappings $\bar{\gamma} : G \to \mathcal{L}$ and $\bar{\mu} : M \to \mathcal{L}$, we see there exists two types of concepts: *object concepts* and *attribute concepts*.

Definition 2.8 (Object Concept/Attribute Concept): Let $g^{\uparrow}$ be the object intent $\{m \in M\} \mid gIm\}$ given $g \in G$, and $m^{\downarrow}$ be the attribute extent $\{g \in G \mid gIm\}$ given $m \in M$. The object concept $(g^{\uparrow\downarrow}, g^{\uparrow})$ is written as $\gamma\{g\}$ and the attribute concept $(m^{\downarrow}, m^{\downarrow\uparrow})$ as $\mu\{m\}$. Given the condition: $gIm \Leftrightarrow \gamma\{g\} \leq \mu\{m\}$ we can also assert $A = \{g \in G \mid \gamma\{g\} \leq (A, B)\}$, and $B = \{m \in M \mid (A, B) \geq \mu\{m\}\}$

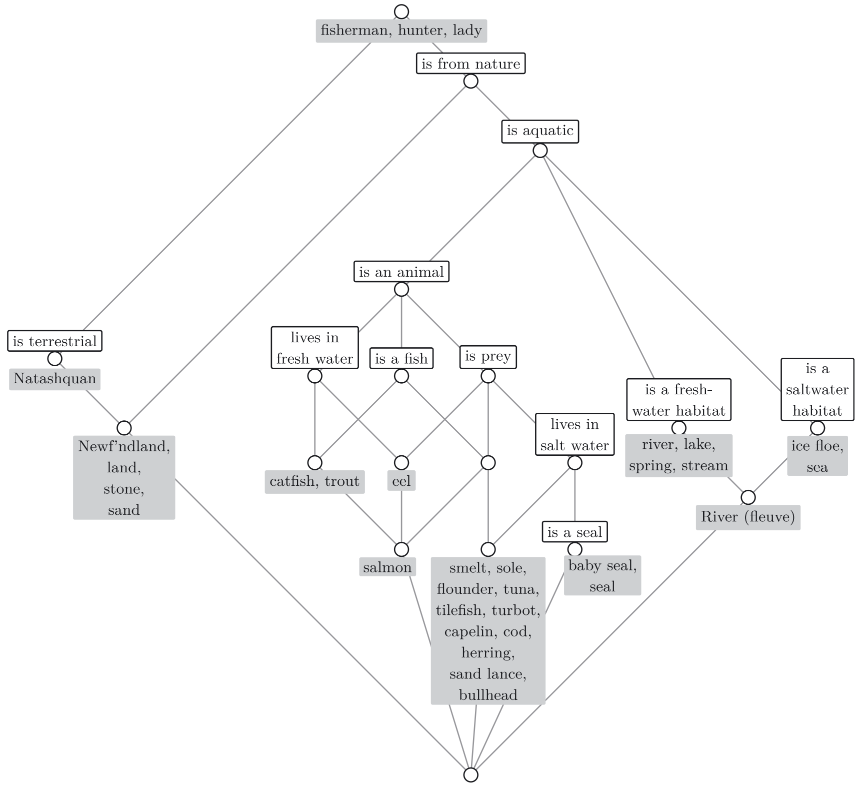


Figure 6: Hasse diagram of $\underline{\mathfrak{B}}\,(\mathbb{K}_2)$.

There is then some relation between what Hébert (2007: 166) calls monoclassification and polyclassification within his finite rooted trees to object concepts and attribute concepts within a complete concept lattice. Consider the following concepts from $\underline{\mathfrak{B}}\,(\mathbb{K}_2)$ (Figure 6):

$$\gamma\{\text{Newf'ndland}\} := \left(\begin{Bmatrix}\text{Newf'ndland, land,}\\ \text{stone, sand}\end{Bmatrix}, \{\text{is terrestrial, is from nature}\}\right), \quad (14)$$

$$\mu\{\text{is terrestrial}\} := \left(\begin{Bmatrix}\text{Natashquan, Newf'ndland,}\\ \text{land, stone, sand}\end{Bmatrix}, \{\text{is terrestrial}\}\right). \quad (15)$$

What Hébert calls monoclassifications are concepts in the form $\mu\{m\}{:=}(m^{\downarrow}, m^{\downarrow\uparrow})$ that have the smallest (empty) intents and largest extents: e.g., $\mu\{\text{is an actant}\}$ in $\underline{\mathfrak{B}}\,(\mathbb{K}_1)$ and $\mu\{\text{is from nature}\}, \mu\{\text{is terrestrial}\} \in \underline{\mathfrak{B}}\,(\mathbb{K}_2)$. Duly, polyclassifications are any concepts that are subconcepts of these superconcepts such that $|A^{\uparrow}| > 1$ for a concept in the form (A, B).

In the case of the concepts γ\{Newf'ndland\}, γ\{land\}, γ\{stone\}, γ\{sand\}, we also have what Hébert (2007: 169) identifies as the isotopy /mineral/ given their intent has the attributes {is terrestrial, is from nature}. Similarly, what Hébert (2007: 169) identifies in << *Quelle affaire!*>> as the significance of the text "lady walking on the ice floe" as a mediating term to the oppositional semes /land/ and /sea/ is expressed in $\underline{\mathfrak{B}}$ $(\mathbb{K}_2)$ through the superconcept/subconcept relation of μ\{is from nature\} $\leq (G, G^{\uparrow})$, where {land, sea} $\in \gamma$\{is from nature\}, and {lady} is an element of the concept with the largest extent $(G, G^{\uparrow})$, or the supremum.

2.1.1 Nested lattices

Hébert also identifies in Vigneault's poem the potential of classification to compare human predation and seal predation, and the parallel "between the recursivity of predations (man hunts the seal who hunts the fish) and the recursivity in the transmission of information (fish A tells B who tells C, 'and so on down the line')" (Hébert 2007: 169). In order to examine this aspect within the context of FCA we can firstly look more closely at the relation between the sub-lattices $\underline{\mathfrak{B}}$ $(\mathbb{K}_1)$ and $\underline{\mathfrak{B}}$ $(\mathbb{K}_2)$ in terms of G, and in respect to the complete lattice formed from $\mathbb{K}_\sigma$. This is achieved this through a *tensor product* of $\underline{\mathfrak{B}}$ $(\mathbb{K}_1)$ and $\underline{\mathfrak{B}}$ $(\mathbb{K}_1)$ which Ganter and Wille (1999) proved is isomorphic to a *direct product* of formal contexts, which is to say more generally that:

$$\underline{\mathfrak{B}}\left(\prod_{t\in T}\mathbb{K}_t\right) \cong \underset{t\in T}{\otimes}\underline{\mathfrak{B}}\ (\mathbb{K}_t). \tag{16}$$

Definition 2.9 (Direct Product): The direct product of two formal contexts, $\mathbb{K}_1 := (G_1, M_1, I)$ and $\mathbb{K}_2 := (G_2, M_2, I_2)$ produces the formal context $\mathbb{K}_1 \times \mathbb{K}_2 := (G_1 \times G_2, M_1 \times M_2, \bigtriangledown)$ in which $(g_1, g_2) \bigtriangledown (m_1, m_2) :\Leftrightarrow ((g_1, m_1) \in I_1 \wedge (g_2, m_2) \in I_2)$.

Definition 2.10 (Tensor Product): The tensor product of two complete lattices $\mathcal{L}_1$ and $\mathcal{L}_2$ is the formal concept lattice of the direct product of their contexts, that is, $\mathcal{L}_1 \otimes \mathcal{L}_2 := \underline{\mathfrak{B}}\ (\mathcal{L}_1 \times \mathcal{L}_2, \mathcal{L}_1 \times \mathcal{L}_2, \bigtriangledown)$ such that $(x_1, x_2) \bigtriangledown (y_1, y_2) :\Leftrightarrow (x_1, \leq y_1) \wedge (x_2 \leq y_2)$.

In Figure 7 we give the Hasse diagram of the tensor product of the two complete sublattices of $\underline{\mathfrak{B}}$ $(\mathbb{K}_\sigma)$ such that the diagram expresses $\underline{\mathfrak{B}}$ $(G, M_1, I \cap G \times M_1)$ as the outer lattice, for which each vertex contains $\underline{\mathfrak{B}}$ $(G, M_2, I \cap G \times M_2)$. In order to show which objects and their associated intents in $\mathbb{K}_2$ intersect with $\mathbb{K}_1$, we shade these nodes and omit labels (c.f. with Figure 6).

The resultant tensor product of sublattices gives a clear indication as to the nature of Hébert's oppositional construct of finite rooted trees based on the nature/

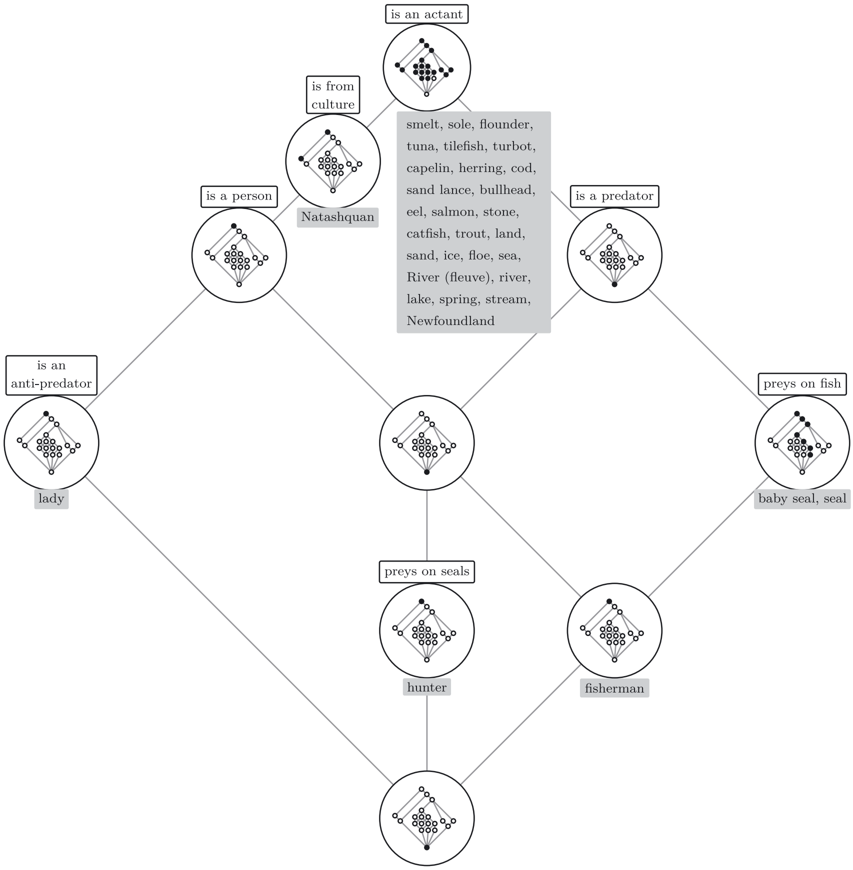


Figure 7: $\underline{\mathfrak{B}}\,(\mathbb{K}_1) \otimes \underline{\mathfrak{B}}\,(\mathbb{K}_2)$.

culture distinction and the recursivity of predations. We can see this firstly through the fact that the intent (colored vertices) of the object subset {baby seal, seal} (//nature//) in $\underline{\mathfrak{B}}\,(\mathbb{K}_1)$ is part of the extent of the concept μ{preys on fish} in $\underline{\mathfrak{B}}\,(\mathbb{K}_2)$, and that this concept has the object {fisherman} (//culture//) in its extent. This gives the relation: γ{fisherman} $\leq$ μ{preys on fish}, which further shows that both γ{fisherman} and γ{baby seal, seal} have the attribute {is a predator} in their intents. We also see from $\underline{\mathfrak{B}}\,(\mathbb{K}_2)$ and the vertices in $\underline{\mathfrak{B}}\,(\mathbb{K}_1) \otimes \underline{\mathfrak{B}}\,(\mathbb{K}_2)$ described by the attributes {is an actant} and {preys on fish}, that both the objects {baby seal, seal} and the subset of objects that contains the species of fish, {smelt, sole, …, bullhead}, intersect in their intent at {is prey, is an animal, is aquatic, is from nature}. While in $\underline{\mathfrak{B}}\,(\mathbb{K}_2)$ both these object subsets are equal in terms of $|\{g\}^{\uparrow}|$ (the cardinality of their intents), within $\underline{\mathfrak{B}}(\mathbb{K}_1) \otimes \underline{\mathfrak{B}}(\mathbb{K}_2)$ because {smelt, sole, … ,bullhead}$^{\uparrow}$ = {is an actant}, and {baby seal, seal}$^{\uparrow}$ = {preys on fish, is a predator, is an actant}, we can think of γ{baby seal, seal} as more *specific*, and thus a key theme in << *Quelle affaire!*>>.

3 Semic analysis

In addition to FCA's parallels to Hébert's "analysis by classification," we can also use it to visualize aspects of Rastier's (1997) "semic analysis." Rastier first proposed the approach within his theoretical framework of *interpretive semantics*, for which it can be described as the identification in a text or an image of semes and how they form larger collections as isotopies and molecules. Semic analysis thus seeks to determine not only the structures of such isotopies, but how semes relate to each other and iterate themselves to form molecules. Hébert (2020: 144) notes that semic analysis is a second generation synthesis of "European structural semantics as developed in the wake of Bréal and Saussure, then Hjelmslev, Greimas, Coseriu and Pottier."

Hébert develops a particular methodology for semic analysis that allows Rastier's larger theoretical framework to be easily applied to source texts. The method centers on the creation of various types of tables: *heuristic semic tables*, *analytical semic tables* and *comprehensive semic tables*. We focus on the analytical semic table, an example found below:

Signified	Seme /x/	Justification
'signified 1'	inherent	...
'signified 2'	...	...
'signified 3'	afferent	...

(17)

The table is primarily used in order to record actualizations of a given seme in a text that is intrinsic, or of interest for analysis.

If we reduce an analytical semic table further and ignore the Justification column as an annotation, then we have a mapping to a context $\mathbb{K}$ given that semes can be considered as $m \in M$ and signifieds as $g \in G$ for which a Boolean as gIm indicates a signified g has as attribute an m. Consider the context $\mathbb{K}_3$ and a visualization of its complete concept lattice in Figure 9.

The context $\mathbb{K}_3$ is a mapping from an analytical semic table Hébert (2020: 159–160) uses for an application of semic analysis on the isotopy /navigation/ in the

There was a fine ship, carved from solid gold
With azure reaching masts, on seas unknown.
Spread-eagled Venus, naked, hair back thrown,
Stood at the prow. The sun blazed uncontrolled.

But on the treacherous ocean in the gloom
She struck the great reef where the Sirens chant.
Appalling shipwreck plunged her keel aslant
To the Gulf's depths, that unrelenting tomb.

She was a Golden Ship: but there showed through
Translucent sides treasures the blasphemous crew,
Hatred, Disgust and Madness, fought to share.

How much survives after the storm's brief race?
Where is my heart, that empty ship, oh where?
Alas, in Dream's abyss sunk without trace.

Figure 8: The poem *The golden ship* by Émile Nelligan (1960).

poem *The golden ship* from 1899 by Émile Nelligan (Figure 8). Our context $\mathbb{K}_3$ is *clarified*, that is to say that for any object $g, h \in G$ for which $g^{\uparrow} = h^{\uparrow}$, it always follows that $g = h$, and duly $m^{\downarrow} = n^{\downarrow}$ implies $m = n$ such that $(m, n) \in M$. Our clarified context $\mathbb{K}_3$ thus gives object equivalences, that is, signifiers are grouped according to a shared intent.

The concept lattice $\underline{\mathfrak{B}}\ (\mathbb{K}_3)$ describes a structure we associate to the classification of semes Hébert gives regarding the isotopy /navigation/. Consider the following six formal concepts as significant semic structures that delineate intent and extent-relative concepts:

$$\mathfrak{c}_1 := (G, \{/\text{navigation}/\}), \tag{18}$$

$$\mathfrak{c}_2 := \left(\left\{\begin{array}{l}\text{vint, azure, treasures,}\\ \text{Gulf, depths, Cypine}\end{array}\right\}, \left\{\begin{array}{l}\text{actualized seme,}\\ \text{afferent seme,}\\ /\text{navigation}/\end{array}\right\}\right), \tag{19}$$

$$\mathfrak{c}_3 := \left(\left\{\begin{array}{l}\text{sunk, saliors, masts, seas,}\\ \text{prow, reef, ocean, shipwreck,}\\ \text{keel, Ship, flancs, ship, siren,}\\ \text{tempête}\}, \left\{\begin{array}{l}\text{actualized seme,}\\ \text{inherent seme,}\\ /\text{navigation}/\end{array}\right\}\end{array}\right.\right), \tag{20}$$

$$\mathfrak{c}_4 := \left(\{G \setminus \{\text{abyss}\}\}, \left\{\begin{array}{l}\text{actualized seme,}\\ /\text{navigation}/\end{array}\right\}\right), \tag{21}$$

$$\mathfrak{c}_5 := (\{\text{abyss}\}, \{/\text{navigation}/\}), \tag{22}$$

$$\mathfrak{c}_6 := (\emptyset, M). \tag{23}$$

The largest concept $\mathfrak{c}_1$ is the isotopy itself as a descriptor of all objects (signifiers), with subconcepts below it that contain various subsets, while the smallest concept $\mathfrak{c}_5$ contains the single object {abyss} such that $\mathfrak{c}_5 \leq \mathfrak{c}_1$. The lattice provides what Greimas and Courtés (1979: 163) identify as the minimal context for an isotopy through a "joining together of at least two semic figures." In $\underline{\mathfrak{B}}(\mathbb{K}_3)$ these figures are embodied in the afferent/inherent semic axis of $\mathfrak{c}_2 - \mathfrak{c}_3$ and their intersection at {actualized seme, /navigation/}, which is what Hébert (2020: 146) notes as the "activation" of the context given there are no neutralizations or virtualizations present in the poem.

3.1 Preconcept lattices

We can further consider the example of the generalization of formal concepts as *preconcepts*. These smaller structures are subconcepts, and allow for the identification of the smallest semantic units associated to sentences (line combinations). Burgmann and Wille (2006: 80) first described *preconcepts* as a means in which to "mathematize the notion of a 'preconcept' which is used in Piaget's cognitive psychology to explain the developmental stage between the stage of senso-motoric intelligence and the stage of operational intelligence."

Definition 3.1 (Preconcept lattice): We define a preconcept of a formal context (G, M, I) as the pair (A, B) such that $A \subseteq G$, and $B \subseteq M$, and $A \subseteq B^{\downarrow}(\Leftrightarrow A^{\uparrow} \supseteq B)$, which is thus a generalization of the definition of a formal concept given in Definition 2.6. The set of all preconcepts of a formal context (G, M, I) is denoted by $\mathfrak{V}(G, M, I)$. Preconcepts are naturally ordered through

$$(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 \wedge B_1 \supseteq B_2, \tag{24}$$

for which the ordered set of $\underline{\mathfrak{V}}(G, M, I) := (\mathfrak{V}(G, M, I), \leq)$ is a complete lattice called the preconcept lattice such that:

$$\bigwedge_{t \in T} (A_t, B_t) = \left(\bigcap_{t \in t} A_t, \bigcup_{t \in T} B_t \right) \text{ and } \bigvee_{t \in T} (A_t, B_t) = \left(\bigcup_{t \in t} A_t, \bigcap_{t \in T} B_t \right), \tag{25}$$

for all $(A_t, B_t) \in \mathfrak{V}(G, M, I)\,(t \in T)$.

Given that preconcepts are smaller units of meaning according to the condition $A \subseteq B^{\downarrow}(\Leftrightarrow A^{\uparrow} \supseteq B)$, we firstly apply an operator $\mathbb{V}$ on $\mathbb{K}_3$ from *The golden ship* in order to generate all preconcepts. We call $\mathbb{V}(\mathbb{K}))$ a *derived* formal context such that:

$$\mathbb{V}(\mathbb{K}) := (G \dot{\cup} M, G \dot{\cup} M, I \cup (\neq \setminus G \times M). \tag{26}$$

Wille (2004: 5) has previously proved that

$$\underline{\mathfrak{B}}\,(G, M, I) \cong \underline{\mathfrak{B}}\,(G \dot{\cup} M, G \dot{\cup} M, I \cup (\neq \setminus G \times M)). \tag{27}$$

That is, a complete protoconcept lattice is isomorphic to the lattice of a derived context.

Consider then the derived context $\mathbb{V}(\mathbb{K}_3)$ we provide in Figure 11. What we will call the *atoms* of its corresponding preconcept lattice $\underline{\mathfrak{B}}\,(\mathbb{V}(\mathbb{K}_3))$ given in Figure 10 are defined more generally as the preconcepts $(\varnothing, M \setminus \{m\})$ such that $m \in M$ as well as the preconcepts $(\{g\}, M)$ such that $g \in G$ and $\{g\}^{\uparrow} = M$. The *coatoms* are duly the preconcepts $(G \setminus \{g\}, \varnothing)$ such that $g \in G$ as well as the preconcepts $(G, \{m\})$ such that $m \in M$ and $\{m\}^{\downarrow} = G$ (Burgmann and Wille 2006: 81). We call the set of all atoms in a complete lattice $\mathcal{J}$ and the set of all coatoms $\mathcal{M}$.

There exist preconcepts that are the images of subsets of $\mathcal{J}$ and $\mathcal{M}$ because atoms and coatoms are described through set complements. We indicate these images through the condition on a complete lattice L, where a is an element of $A \subseteq \mathcal{J}(L)$, and b is an element of $B \subseteq \mathcal{M}(L)$ such that:

$$\nwarrow a := \bigvee\{x \in L \mid x \ngeq a\}, \tag{28}$$

$$\swarrow b := \bigvee\{x \in L \mid x \nleq b\}. \tag{29}$$

Here, a *prototypic object* that we denote as $\nwarrow a$, is the image of a coatom, or the set complement of a single attribute in M, while a *characteristic attribute* as $\swarrow b$, is an image of a set complement of a single object in G. Semantically we can think of these elements as the Boolean operations of *negation* "$\neg$" on a formal concept, or an *opposition* "$\llcorner$" (Wille 2004: 2–3):

$$\neg(A, B) := (G \setminus A, (G, \setminus A)^{\uparrow}), \tag{30}$$

$$\lrcorner(A, B) := ((M \setminus B)^{\downarrow}, M, \setminus B). \tag{31}$$

For example, the concept formed from the attribute '$\swarrow$abyss' in $\underline{\mathfrak{B}}\,(\mathbb{V}(\mathbb{K}_3))$ is equivalent to what Greimas and Courtés (1979: 60–61) call *contradiction*. Here, they describe ¬/abyss/⟨···⟩/abyss/ as an assignment of the binary present/absent. Regarding $\nwarrow$-images, we use the example of '$\nwarrow$afferent seme,' which is an opposition of the afferent seme type. Here an apposition is equivalent to what Hébert (2020: 146) describes as the *virtualization* of a seme: that is, an opposition to its actualization.

But from these conditions we also have subpreconcepts which we understand as representing the smallest units of meaning. Consider the first sentence across lines 1–2 of *The golden ship* in light of Hébert's isotopy /navigation/:

There was a fine ship, carved from solid gold

With azure reaching masts, on seas unknown.

Let the set R:=coloneq {azure, ship, masts, seas} be a collection of signifieds in G of which we gave in $\mathbb{K}_3$ (Figure 9a), and which we associate to lines 1–2 of the poem. In $\underline{\mathfrak{B}}\,(\mathbb{V}\,(\mathbb{K}_3))$, there exists a subpreconcept that we label $\mathfrak{p}_1$ in Figure 10 such that

$$\mathfrak{p}_1 := (\left\{\begin{array}{l} G \setminus \{\text{abyss}\}, \\ \nwarrow\text{inherent seme}, \\ \nwarrow\text{afferent seme} \end{array}\right\}, \left\{\begin{array}{l} /\text{navigation}/, \swarrow\text{abyss}, \\ \text{actualized seme} \end{array}\right\}). \tag{32}$$

R is embedded in $\mathfrak{p}_1$, such that its intent is the subset {actualized seme, $\swarrow$abyss, /navigation/}. As the smallest preconcept formed through R, $\mathfrak{p}_1$ is a coatom that semantically implies a type of negation of the signified 'abyss' in addition to pertaining to the features of an {actualized seme} and the signified as an element of the seme as isotopy /navigation/ (Figure 10). It is a subpreconcept to the preconcepts formed through the attributes {actualized seme}, { $\swarrow$abyss}, {/navigation/}. This shows the isotopy /navigation/ contains two large concepts μ{$\swarrow$abyss} and μ{actualized seme} as features or *preconceptual* planes (Figure 11).

3.2 Inter- and intra-semiotic isotopies

Hébert (2020: 163) also presents semic analysis through reduced analytical semic tables for tracking isotopies that occur within Réne Magritte's 1930 oil painting *La clef des songes* (*Key of Dreams*). Magritte's painting (Figure 12) is set out in a 6 × 2 grid in which various commonly encountered objects are depicted with labels that describe other objects that are seemingly unconnected. Hébert utilizes the example as a means to distinguish between the notion of *intra-semiotic* repetition, that is, when a molecule or semic iteration is for example limited to a text, to that of *inter-semiotic* repetition, or when a molecule or isotopy can be found activated both within a text and an accompanying image.

In order to explore these notions in regards to FCA, we provide a map of Hébert's semic table to a pair of complete contexts visualized in Figures 13 and 14. These ordered concept lattices demarcate Magritte's *La clef des songes* into formal concepts

	actualized seme	inherent seme	afferent seme	/navigation/
abyss				×
vint, azure, treasures, Gulf, depths, Cypine	×		×	×
sunk, saliors, masts, seas, prow, reef, ocean, shipwreck, keel, Ship, flancs, ship, siren, tempête	×	×		×

(a) Clarified context $\mathbb{K}_3$.

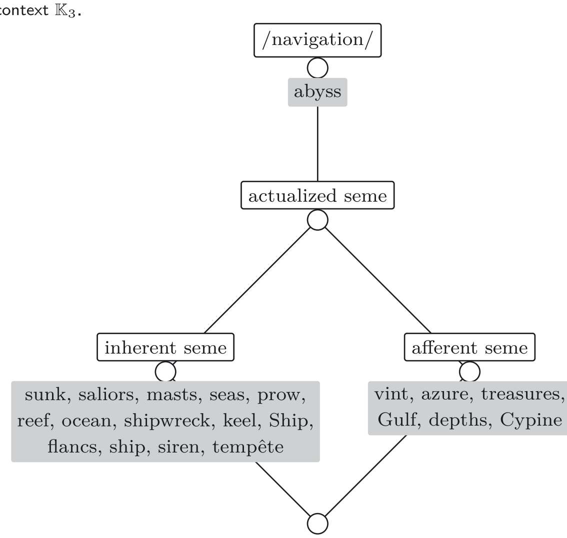


Figure 9: Formal context $\mathbb{K}_3$ and corresponding Hasse diagram $\underline{\mathfrak{B}}(\mathbb{K}_3)$ of Hébert's semic analysis of the isotopy /navigation/ from Émile Nelligan's *The golden ship*.

after Hébert's table for which $\underline{\mathfrak{B}}(\mathbb{K}_4)$ maps images (tokens), and $\underline{\mathfrak{B}}(\mathbb{K}_5)$ maps words (signifiers).

The Hasse diagrams provide a way in which to quickly locate intra-semiotic isotopies as semic molecules given that according to the definition by Rastier (2016: 499–500), a molecule is simply "a stable grouping of semantic features" that is structured in some way through a collection of semes. As an example consider the following attribute concept in the form $(B^{\downarrow}, B)$ from $\underline{\mathfrak{B}}(\mathbb{K}_4)$ (Figure 13):

$$(\{\text{Image 3 (hat), Image 2 (shoe)}\}, \{\text{/water protection/, /black/, /clothing/}\}), \qquad (33)$$

which we equate to the semic molecule /water protection/ + /black/ + /clothing/ through its tokens, the images of a hat and a shoe in *La clef des songes*. The intent of these two tokens yields:

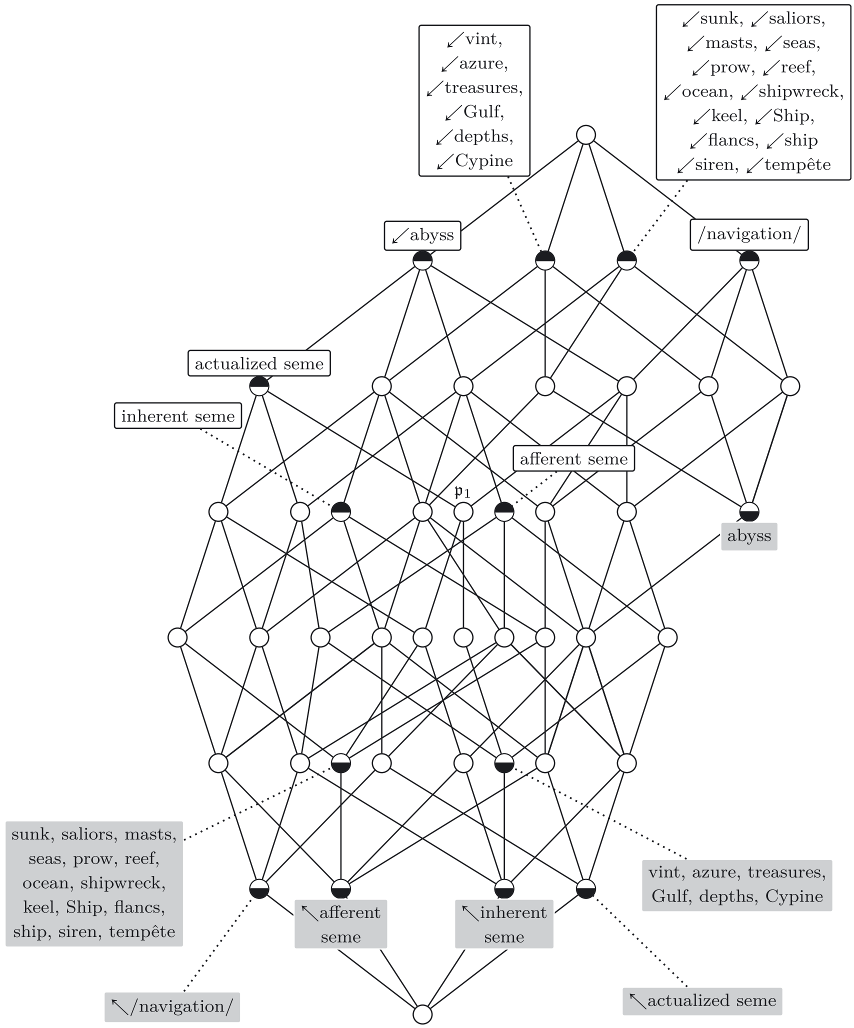


Figure 10: Hasse diagram of the preconcept lattice $\underline{\mathfrak{B}}\,(\mathbb{V}(\mathbb{K}_3))$.

$$(\{\text{Image 3 (hat)}, \text{Image 2 (shoe)}\})^{\uparrow} := \left\{\begin{array}{l} /\text{water protection}/, /\text{black}/, \\ /\text{clothing}/, /\text{hot}/, /\text{familiar}/, \\ /\text{solid}/, /\text{low}/, /\text{curved}/, \\ /\text{container}/, /\text{liquid}/, /\text{culture}/, \\ /\text{inedible}/, /\text{protective}/, \\ /\text{concrete}/, /\text{inanimate}/ \end{array}\right\}. \tag{34}$$

Similarly in $\underline{\mathfrak{B}}\,(\mathbb{K}_5)$ (Figure 14) we have:

	actualized seme	inherent seme	afferent seme	/navigation/	↙abyss	↙vint, ↙azure, ↙treasures, ↙Gulf, ↙depths, ↙Cypine	↙sunk, ↙saliors, ↙masts, ↙seas, ↙prow, ↙reef, ↙ocean, ↙shipwreck, ↙keel, ↙Ship, ↙flancs, ↙ship, ↙siren, ↙tempête
↖actualized seme		×	×	×	×	×	×
↖inherent seme	×		×	×	×	×	×
↖afferent seme	×	×		×	×	×	×
↖/navigation/	×	×	×		×	×	×
abyss				×		×	×
vint, azure, treasures, Gulf, depths, Cypine	×		×	×	×		×
sunk, saliors, masts, seas, prow, reef, ocean, shipwreck, keel, Ship, flancs, ship, siren, tempête	×	×		×	×	×	

Figure 11: The *derived* formal context of $\mathbb{V}(\mathbb{K}_3)$).

$$(\{\text{Word 4“ ceiling”, Word 6“ desert”}\}, \{\text{/container/, /hot/}\}), \tag{35}$$

with the following intent:

$$(\{\text{Word 4“ ceiling”, Word 6“ desert”}\})^{\uparrow} := \left\{ \begin{array}{l} \text{/container/, /hot/, /low/,} \\ \text{/solid/, /NA/, /straight/,} \\ \text{/inanimate/, /nature/,} \\ \text{/brightness/, /concrete/} \end{array} \right\}), \tag{36}$$

as the semic molecule /container/ + /hot/ with signifiers “ceiling” (*la Plafond*) and “desert” (*le Désert*). But given we also have object concepts in the form $(A, A^{\uparrow})$ such as found in (34) and (36) means that for large formal contexts with large intents, we would ideally like to identify subsets of M that are informed through a collection of primary attributes that are *dependent* on a secondary attribute(s). This would allow for a stricter definition of Rastier’s notion of the *stability* of a semic molecule through a reduction in the attribute set via a distinction between $\mu\{m\}$ concepts that contain equivalences (i.e., $m \cong m' \mid (m, m') \in M$), and $\gamma\{g\}$ concepts whose intents may not readily describe *complete* semic molecules – i.e., there may be other tokens or signifiers that satisfy parts of the attribute subset.

Figure 12: René Magritte *La clef des songes* (*Key of Dreams*), oil on canvas, 1930. The work comprises of 6 images (tokens) each accompanied by a word (signifier). Image 1 (top left): egg; word 1: *l'Acacia* (Acacia). Image 2 (top right): shoe; word 2: *la Lune* (moon). Image 3 (middle left): hat; word 3: *la Neige* (snow). Image 4 (middle right): candle; word 4: *le Plafond* (ceiling). Image 5 (bottom left): glass; word 5: *l'Orage* (storm). Image 6 (bottom right): hammer; word 6: *le Désert* (desert).

3.3 Attribute dependency formulas

In order to explore inter-semiotic isotopies between $\underline{\mathfrak{B}}(\mathbb{K}_4)$ and $\underline{\mathfrak{B}}(\mathbb{K}_5)$ we introduce Bělohlévek and Sklenář's (2005) *attribute-dependency formulas* as a method for rule-based attribute reduction and concept exploration within each lattice.

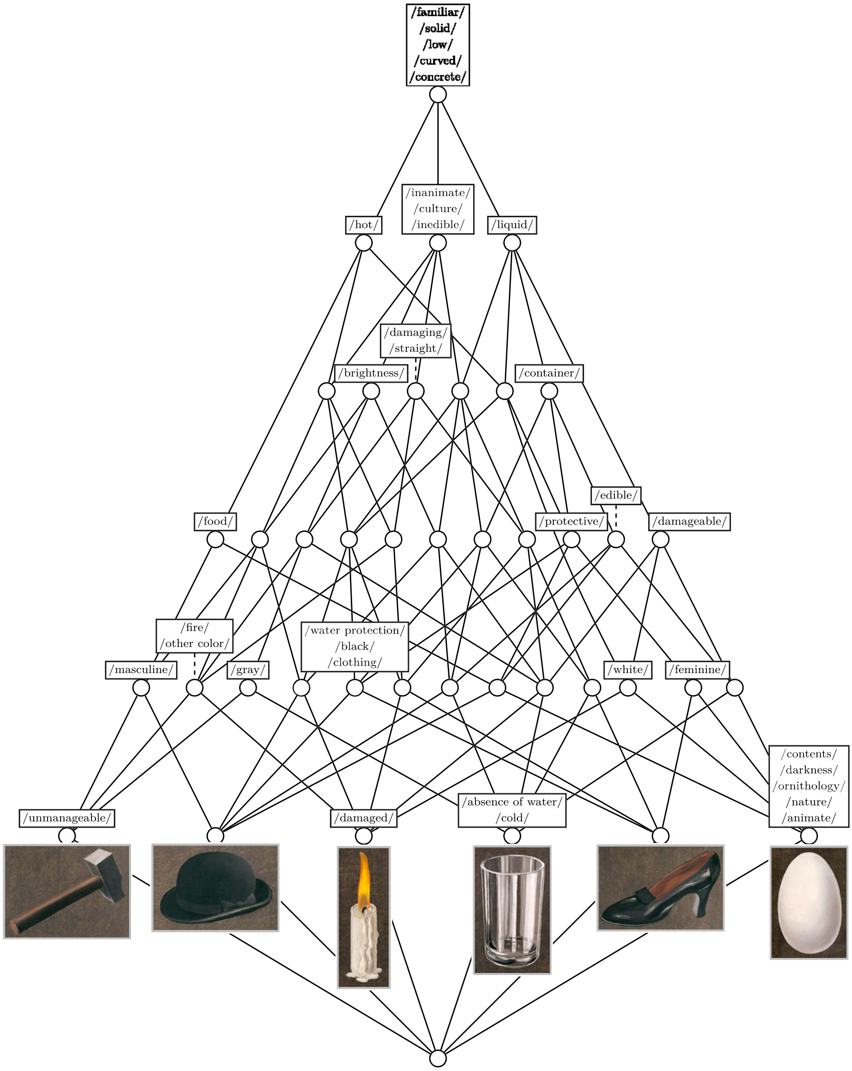


Figure 13: Hasse diagram of $\underline{\mathfrak{B}}\,(\mathbb{K}_4)$.

Definition 3.2 (Attribute-dependency formula): A formal concept (A, B) in (G, M, I) satisfies an attribute dependency formula (ADF), φ in the form $m \sqsubseteq m_1 \sqcup \cdots \sqcup m_n$ if $m, \ldots, m_n \in B$. We let m be the secondary attribute and $m_1, \ldots, m_n$ be primary attributes. Given a formal concept $(A, B) \in \underline{\mathfrak{B}}\,(G, M, I)$, we denote $(A, B) \models \varphi$ to mean that (A, B) satisfies φ.

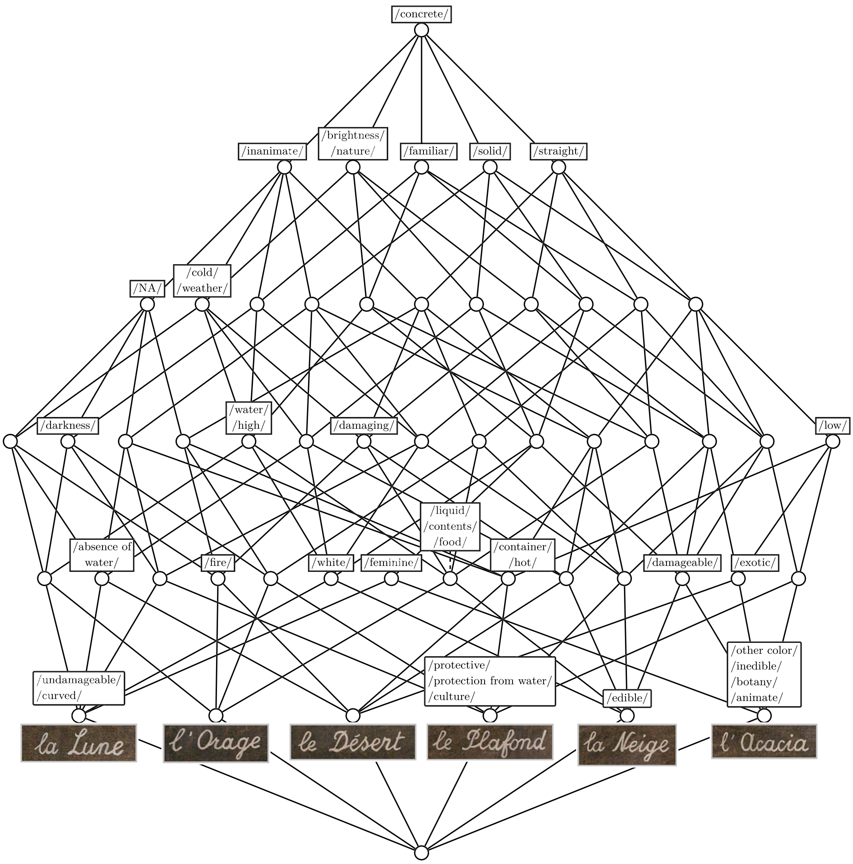


Figure 14: Hasse diagram of $\underline{\mathfrak{B}}\ (\mathbb{K}_5)$.

Consider the set $\mathcal{C} \coloneqq \{\varphi_1, \varphi_2\}$ as a subset of all AFDs, in which we associate φ_1 to $\underline{\mathfrak{B}}\ (\mathbb{K}_4)$, and φ_2 to $\underline{\mathfrak{B}}\ (\mathbb{K}_5)$ such that:

$$\varphi_1 :=/\text{white}/ \sqsubseteq (/\text{hot}/ \sqcup /\text{liquid}/) \sqcup (/\text{liquid}/ \sqcap /\text{damgeable}/), \tag{37}$$

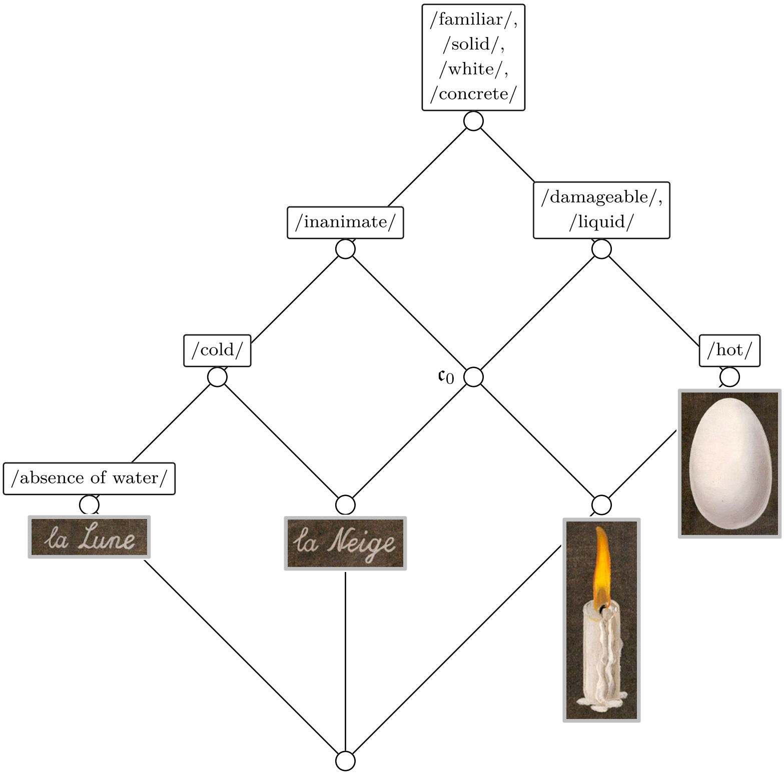
$$\varphi_2 :=/\text{white}/ \sqsubseteq (/\text{absence of water}/ \sqcup /\text{cold}/ \sqcup /\text{inanimate}/)\sqcup \tag{38}$$

$$(/\text{solid}/ \sqcap /\text{familiar}/). \tag{39}$$

Here, we will to focus on the seme /white/ and the nominated dependent semes as they associate to the tokens (objects) {Image 1 (egg)} and {Image 4 (candle)} in $\underline{\mathfrak{B}}\ (\mathbb{K}_4)$ as well as signifiers {Word 3 'snow' *la Neige*} and {Word 2 'moon' *la Lune*} in $\underline{\mathfrak{B}}\ (\mathbb{K}_5)$. As Bělohlávek and Sklenář (2005: 180) note, in general, we can consider expressions

	/absence of water/	/cold/	/inanimate/	/familiar/, /solid/, /white/, /concrete/	/damageable/, /liquid/	/hot/
Word 2: 'moon' *la Lune*	×	×	×	×		
Word 3: 'snow' *la Neige*		×	×	×	×	
Image 1: (egg)				×	×	×
Image 4: (candle)			×	×	×	×

(a) The context $\mathbb{K}_{\theta_{\mathcal{C}}}$ as the subposition of $\mathbb{K}_{\varphi_1}$ and $\mathbb{K}_{\varphi_2}$.

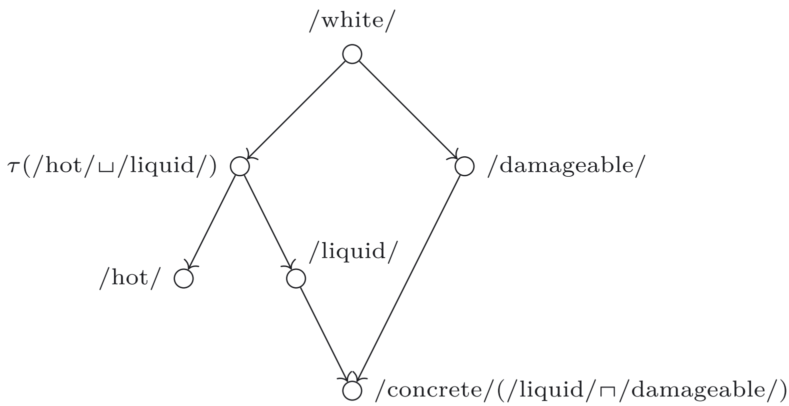


(b) Lattice of ordered concepts $\mathfrak{B}(\mathbb{K}_{\theta_{\mathcal{C}}})$.

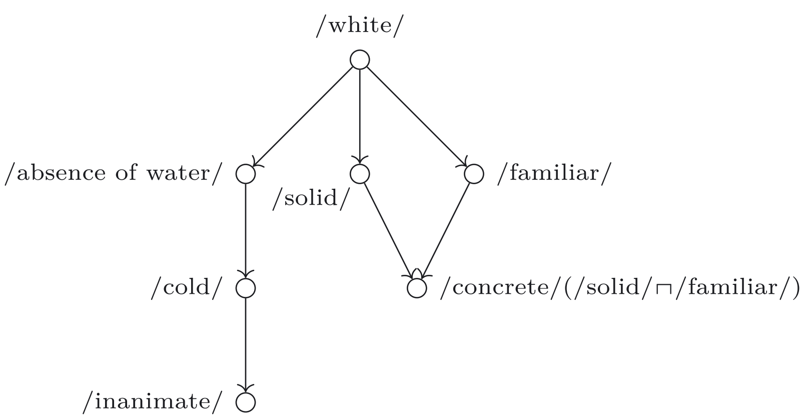
Figure 15: Formal context $\mathbb{K}_{\theta_C}$ and corresponding Hasse diagram $\mathfrak{B}$ $(\mathbb{K}_{\theta_C})$.

within ADFs as *terms* over B in the form $y \sqsubseteq t(y_1, \ldots y_n)$. Here, we designate each attribute $m \in M$ as a term, in addition to identifying terms through Booleans such as $(t_1 \sqcup t_2)$. For example, we see in (38) that (/solid/ ⊓/familiar/) can be expressed as/concrete/(/solid/ ⊓/familiar/) given {/solid/, /familiar/}$^{\uparrow}$ = {/concrete/}.

Because the seme (attribute) /white/ is common to both concept lattices, we use the ADFs above in order to construct a dependency, and thus seek out concepts in both lattices for which given the seme /white/, we also expect in φ_1 the molecule /concrete/(/liquid/ ⊓/damageable/) as well as the molecule /concrete/(/solid/ ⊓/familiar/) in φ_2. We can then express the ADFs as the finite rooted trees $T(\varphi_1)$:and $T(\varphi_2)$:



(40)



(41)

We denote the formal contexts built from these trees as $\mathbb{K}_{\varphi_1}$ and $\mathbb{K}_{\varphi_2}$ whose attributes are subsets of $\mathbb{K}_4 + \mathbb{K}_5$. Here we mean that for each term t in $\mathcal{C}$ there exists a corresponding attribute m in $(M_4 \cup M_5)$. In order to model concepts that are associated to both tokens and signifiers in *Key to Dreams*, we glue these two subcontexts through a *subposition*.

Definition 3.3 (Subposition): Two formal contexts, $\mathbb{K}_1 := (G_1, M, I_1)$ and $\mathbb{K}_2 := (G_2, M, I_2)$, contain the same set of attributes, M, such that $G_1 \cap G_2 = \varnothing$. The subposition of $\mathbb{K}_1$ and $\mathbb{K}_2$ is denoted as $\mathbb{K}_\theta$, such that:

$$\mathbb{K}_\theta := (\dot{G}_1 \cup \dot{G}_2, M, \dot{I}_1 \cup \dot{I}_2), \tag{42}$$

for which $\dot{G}_j := \{j\} \times G_j$ and $\dot{I}_j := \{((j, g), (j, m)) \mid (g, m) \in I_j\}$ for $j \in \{1, 2\}$.

In Figure 15 we give the clarified context $\mathbb{K}_{\theta_C}$ of the subposition of contexts $\mathbb{K}_{\varphi_1}$ and $\mathbb{K}_{\varphi_2}$ as well as its corresponding concept lattice. The lattice shows both semic molecules that are intra-semiotic and inter-semiotic in *Key to Dreams*. Consider the following intra-semiotic concept within the set of signifiers in the form (A, B):

$$\mathfrak{c}_1 := \left(\begin{Bmatrix} \text{Word 2:‘ moon’ } \textit{la Lune}, \\ \text{Word 3:‘ snow’ } \textit{la Neige} \end{Bmatrix}, \begin{Bmatrix} \text{/cold/, /inanimate/,} \\ \text{/familiar/, /solid/,} \\ \text{/white/, /concrete/} \end{Bmatrix}\right), \tag{43}$$

and its corresponding intra-semiotic concept within the set of tokens:

$$\mathfrak{c}_2 := \left(\begin{Bmatrix} \text{Image 4: (candle),} \\ \text{Image 1: (egg)} \end{Bmatrix}, \begin{Bmatrix} \text{/hot/, /liquid/, /damageable/,} \\ \text{/familiar/, /solid/, /white/,} \\ \text{/concrete/} \end{Bmatrix}\right). \tag{44}$$

These two intra-semiotic isotopies are themselves subconcepts to formal concepts formed at the nodes labelled with the attributes {/inanimate/} and {/damageable/, /liquid/} in $\underline{\mathfrak{B}}(\mathbb{K}_{\theta_C})$:

$$\mathfrak{c}_3 := \left(\begin{Bmatrix} \text{Image 4: (candle),} \\ \text{Word 2:‘ moon’ } \textit{la Lune}, \\ \text{Word 3:‘ snow’ } \textit{la Neige} \end{Bmatrix}, \begin{Bmatrix} \text{/inanimate/, /familiar/,} \\ \text{/solid/, /white/, /concrete/} \end{Bmatrix}\right), \tag{45}$$

$$\mathfrak{c}_4 := \left(\begin{Bmatrix} \text{Image 4: (candle),} \\ \text{Image 1: (egg),} \\ \text{Word 3:‘ snow’ } \textit{la Neige} \end{Bmatrix}, \begin{Bmatrix} \text{/liquid/, /damageable/,} \\ \text{/familiar/, /solid/, /white/,} \\ \text{/concrete/} \end{Bmatrix}\right). \tag{46}$$

The formal concepts $\mathfrak{c}_3$ and $\mathfrak{c}_4$ are thus two inter-semiotic molecules as superconcepts ($\mathfrak{c}_1 \le \mathfrak{c}_3$, and $\mathfrak{c}_2 \le \mathfrak{c}_4$) that cover a subset of signifiers and tokens in *Key to Dreams*. These two molecules have a greatest lower bound as the subconcept $\mathfrak{c}_0$ (Figure 15):

$$\mathfrak{c}_0 := \left(\begin{Bmatrix} \text{Image 4: (candle),} \\ \text{Word 3:‘ snow’ } \textit{la Neige} \end{Bmatrix}, \begin{Bmatrix} \text{/inanimate/,} \\ \text{/damageable liquid/,} \\ \text{/familiar/, /solid/,} \\ \text{/white/, /concrete/} \end{Bmatrix}\right), \tag{47}$$

which we contend is the smallest and most generalized expression of the inter-semiotic molecule in *Key to Dreams* obtained through an ADF restriction on the seme {/white/}.

3.4 Attribute implications

The use of ADFs to assist in constructing subconcept/superconcept relations between molecules also points towards how we can parse *implications* from a concept lattice in order to establish /seme/ → /seme/ structures that define a semic molecule's structure.

Definition 3.4 (Attribute Implication): An implication over M is denoted as $X \to Y$, where $X, Y \subseteq M$. An implication that is valid in $\mathbb{K}$ is denoted as $\mathbb{K} \models X \to Y$, if $X^{\downarrow} \subseteq Y^{\downarrow}$, that is, each object in $\mathbb{K}$ that possesses all attributes in X, also possesses all attributes in Y. An implication set, $\mathcal{P}$, is valid in $\mathbb{K}$ if all implications in $\mathcal{P}$ are also valid in $\mathbb{K}$.

Although Hébert does not discuss implications directly within Rastier's framework of interpretive semiotics, Greimas and Courtés (1979: 152) equate implications to *presuppositions*. That is, they are envisaged as "logically prior to implication: the 'if' would not find its 'then,' if the latter did not already exist as the presupposed." Within the framework of FCA, implications are element/subset relations that emerge from the intent of the formal contexts.

For example, in the concept lattice $\underline{\mathfrak{B}}\,(\mathbb{K}_{\theta_C})$ (Figure 15), the two inter-semiotic molecules we identified as $\mathfrak{c}_3$ and $\mathfrak{c}_4$ contain the following attribute (seme) implications respectively:

$$\{\text{/absence of water/}, \text{/cold/}\} \to \{\text{/inanimate/}\}, \tag{48}$$

$$\{\text{/hot/}\} \to \{\text{/damageable/}, \text{/liquid/}\}, \tag{49}$$

such that both sets of implications also imply {/white/}. But we can further cross-reference these implications as they pertain more generally to the lattices $\underline{\mathfrak{B}}\,(\mathbb{K}_4)$ (Figure 13) and $\underline{\mathfrak{B}}\,(\mathbb{K}_5)$ (Figure 14) to find further implications such as:

$$(\{\text{/hot/}\} \to \{\text{/solid/}, \text{/low/}\}) \in \underline{\mathfrak{B}}\,(\mathbb{K}_4), \tag{50}$$

$$(\{\text{/absence of water/}\} \to \{\text{/weather/}, \text{/nature/}\}) \in \underline{\mathfrak{B}}\,(\mathbb{K}_5). \tag{51}$$

We also find from our earlier example in Equation (33) of the semic molecule /water protection/ + /black/ + /clothing/ from the tokens {Image 3 (hat), Image 2 (shoe)} that:

$$\{\text{/water protection/}, \text{/black/}, \text{/clothing/}\} \to \{\text{/container/}, \text{/liquid/}, \text{/hot/}\}. \tag{52}$$

4 Conclusions

In this article we have introduced various mathematical formalisms to describe semiotic structures within the framework of “analysis by classification” and “semic analysis.” We firstly introduced Hébert’s disconnected finite rooted trees for the description of thematized classification in Vigneault’s <<*Quelle affaire!*>>, and by proposing structure-preserving maps to the formal contexts $\mathbb{K}_1$ and $\mathbb{K}_2$ and their ordered concept lattices, we introduced the utility of FCA for visualizing and reasoning on semiotic structures.

We showed in <<*Quelle affaire!*>> how Hébert’s notion of monoclassification equates to superconcepts in the concept lattices $\underline{\mathfrak{B}}\ (\mathbb{K}_1)$ and $\underline{\mathfrak{B}}\ (\mathbb{K}_2)$ in the form $\mu\{m\}$ $:=(m^{\downarrow}, m^{\downarrow\uparrow}\}$ that have small intents (i.e., $|\ A^{\uparrow}\ | = 1$) and large extents. Duly, polyclassifications equate to subconcepts under the condition $|\ A^{\uparrow}\ | > 1$ for any formal concept (A, B). This also led us to utilize the tensor product of lattices to generate $\underline{\mathfrak{B}}\ (\mathbb{K}_1) \otimes \underline{\mathfrak{B}}\ (\mathbb{K}_2)$ to track the recursivity of predations and oppositional dynamics between the semantic classes of //nature// and //culture//. Here we showed that in <<*Quelle affaire!*>>, the intersection between these semantic classes is readily visualized in the nested lattice through the activated concepts γ\{fisherman\} (//culture//) and γ\{baby seal, seal\} (//nature//) given they share an intent that contains the attribute \{is a predator\}.

This led us to further consider the structure of isotopies in *The golden ship* by Émile Nelligan. Here, we used FCA to firstly map Hébert’s analytic semic table of the poem to a formal context $\mathbb{K}_3$ and its accompanying concept lattice. Using the lattice we identified important formal concepts of the isotopy /navigation/, in particular, an afferent/inherent seme axis associated to the concepts $\mathfrak{c}_1$ and $\mathfrak{c}_2$, and their intersection at the attributes \{actualized seme, /navigation/\}, which equates to what Hébert calls an activation within the narrative plane. We then introduced preconcept lattices via the derived formal context $\mathbb{V}\ (\mathbb{K}_3))$ and the condition $A \subseteq B^{\downarrow} (\Leftrightarrow A^{\uparrow} \supseteq B)$, and showed how the notion of negation $(\neg)$ and opposition [*Ieqn*2] within preconcept lattice constructions mirror what Greimas identifies as contradiction and Hébert identifies as virtualization.

Our final application of FCA considered Hébert’s analysis of inter- and intra-semiotic molecules within Réne Magritte’s *La clef des songes* (*Key of Dreams*). We constructed the formal contexts $\mathbb{K}_4$ and $\mathbb{K}_5$ and their accompanying concept lattices from an analytic semic table forwarded by Hébert in order to examine Rastier’s notion of stability of semantic features within a set of tokens (images) and signifiers (words) within *La clef des songes*. In order to formalize the structure of inter- and intra-semiotic molecules within the art work, we introduced ADFs in order to track attribute relations. We then used the concept lattice $\underline{\mathfrak{B}}\ (\mathbb{K}_{\theta_c})$ as a means to visualize both the intent and extent of the two types of molecules built from attribute dependencies of the seme /white/. The concept lattice visualized the molecules both in the domain of the signifiers and tokens as well as their intersection at the objects {Image 4: (candle)} and {Word 3: 'snow' *la Neige*}. The use of ADFs also highlighted how attribute implications within our constructed concept lattices point towards what Greimas and Courtés call presuppositions between semes.

We consider these insights and formalisms as scaffolds to the discourse offered by Rastier and Hébert regarding semic analysis and analysis by classification, which in turn has highlighted how their approach closely aligns with the original aim of the Greimas semiotics project. That is to say that for Greimas, structural semiotics is a method concerned with a rigor and precision that is expressed through a mathematic-like terminology which can generate formalisms as well rules that identify structural equivalences. We have sought to identify with these same concerns in this article through extending these abstractions by drawing directly from the language of mathematics. But contrary to a desire to obfuscate, our motivation has been to provide a syntax for an interdisciplinary methodology that unites important previous contributions in the fields of structural semiotics and lattice theory in the pursuit of a theory of semantics beyond textual linguistics.

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