Educ Stud Math (2013) 82:51–73
DOI 10.1007/s10649-012-9417-7

Semiotic processes in chat-based problem-solving situations

Christof Schreiber

Published online: 14 July 2012

Abstract This article seeks to illustrate the analysis of episodes of chat sessions based on Charles Sanders Peirce's triadic sign relation. The episodes are from a project called "Math-Chat", which is based on the use of mathematical inscriptions in an experimental setting. What is characteristic of this chat setting is that pupils are required to document their attempts at solving mathematical problems as mutual inscriptions in written and graphical form. In order to analyze the outline, as well as the use and development of mutual inscriptions, a suitable instrument of analysis must first be developed. For this purpose, an interactionist approach is combined with a semiotic perspective. Through the incorporation of a semiotic perspective into an empirical study on learning mathematics at primary level, the development and use of such an instrument are demonstrated. In this way, both the development and also the structure of "semiotic process cards" are explained. In conclusion, my findings related to the use of inscriptions in general and the use of inscriptions in primary-classroom problem-solving processes are presented.

Keywords Qualitative research methodology · Semiotics · Peirce · Inscription · Framing · Digital media

1 Introduction

Comparing spoken and written language, Donaldson (1978) notes: "The spoken word […] exists for a brief moment as one element in a tangle of shifting events, […] and then it fades. The written word endures. It is there on the page, distinct, lasting. We may return to it tomorrow" (p. 90). Krummheuer (2000b) refers to the fleetingness of verbal utterances in learning situations and suggests that: "[…] the quick evaporation and the situational uniqueness of verbal accomplishments impedes reflection on such interactive procedures

This study was supported by Müller-Reitz-Stiftung (T009 12245/02) entitled "Math-Chat: pilot study of chat-based creation of mathematical inscriptions among primary pupils" ("Mathe-Chat: Pilotstudie zur Chat-unterstützten Erstellung mathematischer Inskriptionen unter Grundschülern").

C. Schreiber (✉)
Institut für Didaktik der Mathematik und der Informatik, Goethe-Universität Frankfurt (Germany),
Senckenberganlage 11, 60054 Frankfurt, Germany
e-mail: schreiber@math.uni-frankfurt.de

[…]. Complementing such reflections with a written presentation of the result (especially of the work process) seems helpful" (p. 31).

In mathematics in particular, the learning process depends considerably on written-graphical communication.[1] This is due to the fact that the depiction and description of many mathematical operations can be seen as the mathematical idea or procedure itself and do not necessarily have to be understood as its sole representation in the form of a symbol or sign. Writing and written presentations are integral elements of mathematical communication. Pimm (1987) also mentions that mathematics depends on written forms of communication. A written record of ideas and methods of resolution of mathematical problems changes their status, making them both more explicit and negotiable (see also "externalization tenet," of Bruner, 1996, pp. 22–25). There is very little empirical research that examines the importance of a written record of the resolution of mathematical problems in relation to this process.[2]

In order to access the written products of a problem-solving process, I had pupils solve mathematical problems in a specific setting, namely using two tablet PCs that were connected via internet chat. The chat setting offers a new perspective on fundamental questions about the teaching and learning of mathematics. A chat is a form of interaction based on the written word and graphics while also simulating verbal communication through interactivity. In this way, conceptual verbal situations can be made more accessible to analysis through the medial written and graphical representation. Oral communication between participants is not possible, which makes the writing of questions, tips, suggestions, and different approaches necessary in order to communicate with the other participants. We can hereby study theoretical and methodological questions on the solving of mathematical problems by analyzing written elements. It must be noted that this particular setting was chosen only due to its usefulness for the analysis of written portrayal of mathematical problem-solving strategies and is not to be viewed as a suggestion for a particularly innovative setting for teaching and learning. The aim is solely to encourage pupils to engage in written communication, in order to analyze the meaning and importance of this communication in collective problem-solving strategies.[3] But there was a lack of instruments to analyze the written products of the pupils. So, as part of the "Math-Chat" project, a semiotic instrument of analysis was developed, which enables an accurate examination of the mutually produced processes of written problem solving and communication. This instrument also makes the analysis of any verbal utterances of the participants during this process possible. Thus, this instrument was even more versatile than was initially intended.

In the following sections, I firstly describe the technical and organizational requirements and specifications of the project (Section 2); thereafter I elucidate the term "inscription" as used by Latour and Woolgar (1986) (Section 3). Then I present methodological information on the analysis of the interactions (Section 4). In Section 5, the aspects of Charles S. Peirce's semiotics necessary for the development of the instrument of analysis are illustrated (Section 5), after which I describe the "semiotic process cards" that were developed (Section 6) and give a detailed description of the method of analysis based on an example from the chat project (Section 7). In the final section I present my results and findings (8).

[1] Morgan (1998) describes the widespread significance of writing in mathematical learning processes.

[2] For example Meira (1995) "discusses the production and use of matheamtical notations by elementary school students" (p. 87).

[3] A practically orientated development was conducted by Reinhard (2008) as a "Wiki-basierte Lernumgebung zum kooperativen Lernen mit Neuen Medien im Mathematikunterricht der Primarstufe—WiLM@" (Wiki-based Learning Environment for Cooperative Learning with New Media in Primary Mathematics Classroom—WiLM@).

2 The "Math-Chat" project

The main focus of the "Math-Chat" project lay in the examination of the fundamental problem of the written depiction of collective strategies for the solving of mathematical problems in an experimental setting. The research centered on the type of inscription pupils compiled during the mutual problem-solving process, how these inscriptions were used and developed, and what part they played in structuring the problem-solving process.

The fourth-grade pupils who took part in the project used two tablet PCs to communicate during the chat sessions. There were one or two pupils on each side of the setting. Special pens or markers were used to enter information on the screens. The two computers were situated in different locations, connected by a wireless internet connection. No oral communication was possible between the chat participants, which made it necessary to enter questions, tips, methods, and proposals of resolution in written form in order to communicate with the other participants in the joint problem-solving process. The program NetMeeting is used to facilitate the chat settings.[4] This program enables participants to enter data in two different forms: Alphanumeric data entered using the keyboard appear in the "chatbox", the marker (see Fig. 1 for use of terms) is used to enter data on the "whiteboard". The chatbox and the whiteboard appear on screen side by side as two separate windows.

The aim was to induce pupils to communicate the problem-solving process non-orally, in order to investigate the importance of the written product of the collective problem-solving process, as described above. In this way, the chat setting offers a new perspective on a number of fundamental questions concerning the teaching and learning of mathematics. Chatting is a form of interaction that is based on the written word and graphics, but its interactive nature means it also has similarities to oral interaction. Thus, through the medial written form of communication it is possible to gain insight into a conceptually oral situation. Both theoretical and methodological questions on mutually created, written aspects of a mathematical problem-solving process can be examined, since no oral communication between the participants was possible.

The communication in the chatbox is, to use Dürscheid's (2003) terminology, "quasi-synchronous" (p. 44): Messages had to be typed and sent in order for them to be communicated to the other participant. Until they have been sent they are not visible to him/her. Messages are typed into a window and can be edited or deleted by the sender before being sent. Once a message has been sent it is visible in the chat box of all participating computers (Fig. 1, left). The author and time of sending are also visible and it is not possible to make any further changes to the message.

By contrast all communication on the whiteboard area of the screen takes place simultaneously (Fig. 1, right). This means that every operation or action carried out on the whiteboard area of a computer appears simultaneously on the whiteboard of the other participants. Any changes the author of a message makes in this area are also visible. Any graphics are also available to all participants to edit. During the research project "Math-Chat", all operations that were carried out on the computer screens and all utterances spoken by the participants were recorded. Scenes deemed relevant to the research problem were then transcribed, in order to enable a detailed analysis.

[4] NetMeeting is Freeware from Microsoft

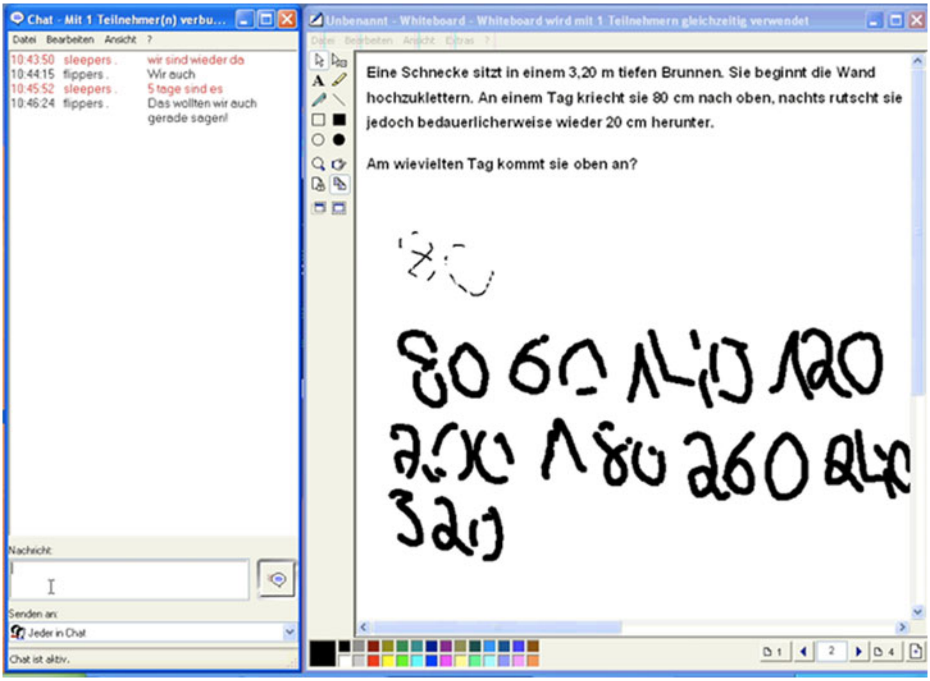


Fig. 1 Screenshot of a "net meeting" (see example in chapter 5)

3 Mathematical inscriptions

In terms of solving problems based on the research of mathematical information and correlations, the participating pupils worked mainly on the written products (of interaction) in the setting described above. I refer to the written-graphical products generated by the pupils in the chat setting as "inscriptions" (Latour & Woolgar 1986). Latour and Woolgar studied the development and evolution of knowledge in laboratories. The different kinds of models, pictures, icons, and notations used in the laboratories are classified by Latour and Woolgar as "inscriptions". They describe several characteristics of inscriptions (Latour & Woolgar, 1986; Latour, 1987):

- Inscriptions are mobile because they are recorded in materials and can be sent by mail, courier, facsimile, or computer networks.
- They are immutable during the process of moving to different places. Inscriptions remain intact and do not change their properties.
- The fact that they can be integrated in publications just after a little cleaning up is described as one of the most important advantages of inscriptions.
- The scale of inscriptions can be modified without changing internal relations.
- It is possible to superimpose several inscriptions of different origins.
- They can be reproduced and spread at low cost in an economical, cognitive, and temporal sense.

Inscriptions are seen by Latour and Woolgar as a very ductile means of representation that is continuously changing and improving. In this way, they represent aspects of the conceptual development during the research process (see also Schreiber, 2004). Latour (1990) talks about “cascades of ever more simplified” (p. 20) inscriptions. Latour and Woolgar's definition of the term “inscription” applies exactly to the subject matter being researched in the “Math-Chat” project. The interest is centered on a detailed analysis of the inscriptional aspect in mathematical interactions, both on the interactive origination of the inscriptions as well as the meaning and importance of the developing inscriptions for the interaction process.

Roth and McGinn (1998) point out that the use of inscriptions is closely interconnected with the social practice in which they originated:

> Inscriptions are pieces of craftwork, constructed in the interest of making things visible for material, rhetorical, institutional, and political purpose. The things made visible in this manner can be registered, talked about, and manipulated. Because the relationship between inscriptions and their referents is the matter of social practice … students need to appropriate the use of inscriptions by participating in related social practices. (p. 54)

Herein lies the basis of our interactionistic approach in relation to the learning of mathematics, which forms the foundation of this project. What is unique about the approach described here is that the focus lies on the process of genesis of individual inscriptions: pupils externalize their ideas in a chat-based dialogue using alphanumeric and/or graphic notations. The reactions of their chat partners enable the gradual development of a single inscription into a joint or mutual inscription. The internet chat method is conducive to this process of text compilation, as it (the process of compilation) becomes both collective and interactive. This process can be viewed as an important component of the (chat-based) interaction and it generates the “taken-as-shared-meaning” of the chat partners (Cobb & Bauersfeld, 1995). There are many other publications that deal with interactively created inscriptions (see Roth & McGinn, 1998; Lehrer, Schauble, & Carpenter, 2000; Sherin, 2000; Meira, 1995, 2002; Gravemeijer, Cobb, Bowers, & Whitenack, 2000; Gravemeijer, 2002; Fetzer, 2007), although all of these focus on face-to-face situations. The focus during this project, however, lay solely on an inscription-based communication between the two sides of the chat setting, which was facilitated by the experimental design.

4 Analysis of interaction

Interaction analysis is a method of analysis that was developed on the basis of the ethno-methodological conversation analysis of Bauersfeld, Krummheuer, and Voigt at the IDM Bielefeld and is concerned with processes of interaction that take place in a school setting. This form of analysis is based on symbolic interactionism:

> The meaning of a thing for a person grows out of the ways in which other persons act toward the person with regard to the thing. Their actions operate to define the thing for the person. Thus, symbolic interactionism sees meaning as social products, as creations that are formed in and through the defining activities of people as they interact. (Blumer, 1969, pp. 4–5)

The meaning of a thing is thereby negotiated through interaction. This negotiation occurs during processes of social interaction from which understanding and cooperation emerge on a semantic level.

In order for this negotiation of meaning to happen, the participants' interpretations of a situation must be adapted to one another. Their definitions of the situation do not necessarily have to be identical, but must harmonize sufficiently to accommodate the continuation and further development of the interaction. In this context, therefore, the product of participants is not thought to be a *shared meaning* but rather a “taken-as-shared-meaning” (Krummheuer & Fetzer, 2005, p. 25). Such an “interim product” of the interaction is generated by the continuing process of negotiation of meaning and signals a thematic openness toward the continuing progress of the interaction (see Naujok, Brandt, & Krummheuer, 2004). Through the interchanging indication of their interpretation, a process of “clearance” takes place during the attribution of meaning of the participants.

Through the interaction analysis, the way in which individuals create and negotiate taken-as-shared-meaning is reconstructed (Krummheuer & Naujok, 1999; Krummheuer, 2000a). The aim is to reconstruct any operations in the situation that are meaningful for the participants and to construct as many interpretations of these actions as possible. These initial interpretations are then reinforced or rejected, in order to ensure as valid an interpretation of the episode as possible.

A number of scenes were selected from excerpts of lessons, which were recorded as screen videos and thereafter transcribed. These scenes were then interpreted in detail using the interaction analysis. Such an analysis is illustrated below as a summarized interpretation and further analyzed using the semiotic aspects described in the following section.

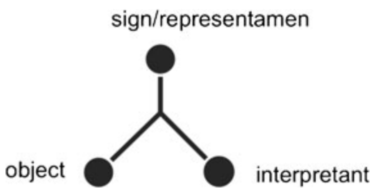
5 Aspects of Charles Sanders Peirce's semiotics

For the analysis of the commonly accomplished inscriptions in the chat-based solving processes, we refer to Peirce's sign model. The Peircean sign model is a very differentiated classification and it is applied by some researchers of didactics of mathematics (e.g. Volkert, 1990; Hoffmann, 1996, 2003; Dörfler, 2004, 2006; Seeger, 2011; Otte, 2011) as well as by pedagogical researchers (i.e. Zellmer, 1979). In comparison to other semiotic approaches (e.g. Saussure and Lacan: see Gravemeijer, 2002), it seems to be a more powerful instrument for my analyses. Among other arguments, the Peircean approach is less oriented towards the language than the ones of Saussure or Lacan (see a detailed discussion by Hoffmann, 2003, pp. 7–12), but it integrates the individual as interpreting instance and opens by this means the option of integrating this approach with an interactional theory of learning and teaching mathematics. Also Eco (1976) points out the advantages of Peircean semiotics in “A Theory of Semiotics” (see the discussion by Eco, 1976, pp. 14–16, where he describes the Peircean approach as more complete and semiotically more fruitful; see also Schreiber, 2004).

5.1 Peirce's triadic sign relation

Peirce's (1931–1935) triadic sign relation consists of a “triple connection of sign, thing signified and cognition produced in the mind” (CP 1.372). The three correlates in this triadic relation can be described as in Fig. 2:

Fig. 2 Peirce's triadic sign relation



> A sign, or representamen, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object. It stands for that object not in all respects, but in reference to a sort of idea, which I have sometimes called the ground of the representamen. (CP 2.228)

That which Peirce (1931–1935) refers to as "sign" or "representamen" can be understood as the external, visually, aurally, or otherwise perceptible depiction of a sign, while the "interpretant" is a sort of inner sign, which is associated with the external perception in the association/view/vision of the observer. The "object" is to be understood as that which the observer of an external sign believes was its creator's purpose. For Peirce, these three correlates are integral parts of a sign and none of the three is superfluous. The sign itself only becomes a sign when it is perceived by an observer to be such. According to Peirce, that which is not interpreted as being a sign is not a sign. (CP 2.228)

Because of the potential confusion in the use of the term "sign" for two of the three correlates (the representamen and the interpretant) and in many cases even for the whole triadic sign relation, I will use the definitions "representamen" and "interpretant" for the correlates and the term "sign triad" for the complete "triple connection" described above. I will retain Peirce's definition of the term "object", but it must be clarified that the term does not necessarily refer to an object in the material sense.

5.2 The foundation of Peirce's triadic sign relation

Applying and developing the Peircean approach, Hoffmann (e.g. Hoffmann, 1996) focuses on the "idea" or "ground" in the Peircean sign model, whereas he calls "das Allgemeine", the "general" (translation by Schreiber), what Peirce calls "ground", or "idea". Hoffmann mentions as examples for the "general": concepts, theories, habits, competences, etc., which are given mentally or physically. The concept of the "general" was in a first attempt fundamental for the analysis of the excerpts of the "Math-Chat" project. Because the interpretant is determined by the concepts, theories, habits, and skills of the observer, the element of generality is the foundation of the triadic sign relation (see also Schreiber, 2005a, 2006). However, the terminology was misleading, so an appropriate term has to be found.

The way in which I use the concept of "general" in my semiotic analyses (Schreiber, 2006), suggests the substitution of this expression with the term "frame", which is used by Goffman (1974, p. 7) with regard to Bateson (1955). For this reason, it will be demonstrated how this "ground of the representamen" is to be integrated into an interactionistic perspective: Each individual creates interpretants against the background of his or her own subjective interpretation experiences and under a specific perspective. Goffman (1974) points out the importance of standardization and the formation of a routine during the "definition of the situation" (p. 1f.) and introduced the term "frame" to describe interpretation processes (Goffman, 1974, p. 7). Krummheuer used the term coined by Goffman in a content-based setting, and related it to curriculum-based educational theory.

The terms "generality" and "frame" correspond largely with one other, each being integrated in their respective theoretical fields. The term preferred for this particular research, the "frame", is well established in interpretative educational research and has been demonstrated to be fitting, as it has proven to be relevant to many areas of reconstructive

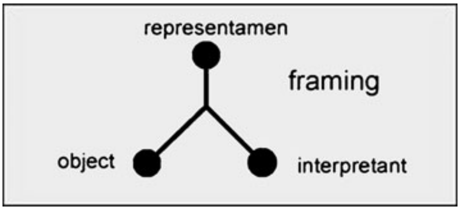


Fig. 3 The frame as the basis of Peirce's triadic sign relation

social research. The term "generality" has otherwise not been further mentioned in literature on semiotics in the didactics of mathematics.

All parts of verbal statements or operations of a given individual in an interaction situation can be identified as familiar representamen. Frames are "activated" by familiar representamen. These framing procedures can be taken as the "ground of representamen", as defined by Peirce (see Fig. 3).

The variance of the interpretant is limited by the frame which is triggered within each individual by the representamen. The representamen does not stand for this object in every regard, but rather only in regard to an activated frame. The connection of Peirce's semiotics with Goffman's frame analysis makes the empirical analysis of frames on the detailed level of Peircean triads possible.

5.3 The chaining process

Peirce describes meaning as a constantly developing process, in which the interpretant of a given triadic sign relation becomes the representamen of a further triad: "Anything which determines something else (its *interpretant*) to refer to an object to which itself refers (its *object*) in the same way, the interpretant becoming in turn a sign, and so on *ad infinitum*" (CP 2.303; italics Peirce's own). Peirce believed that every interpretant within a triad could also be interpreted within another (Fig. 4). This continuous process of semiosis is potentially endless. It cannot be brought to an end but can be interrupted (CP 5.284). Peirce noted that the identification of a first or final sign is not possible in this context.

Saenz Ludlow (2006) also alludes to "unlimited semiosis" (p. 188; see also Eco, 1979, p. 198) and describes an illustration similar to Fig. 4 as "meaning emerging in the translation of signs into new signs" (ibid.). Presmeg (2006) describes and compares this chaining process first based on Saussure's dyadic sign model and later based on Peirce's triadic sign model (p. 165 ff.). She points out, that the "dyadic chaining conceptual model was not completely

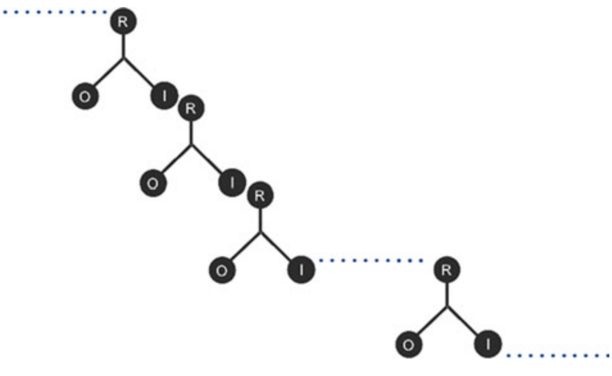


Fig. 4 The unlimited process of semiosis. (Schreiber, 2010, p. 37; similar in Schreiber, 2006, p. 248)

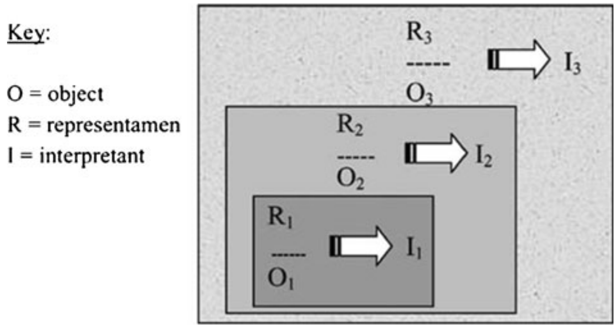


Fig. 5 "A triadic representation of a nested chaining of three signs." (Presmeg, 2006, p. 170)

adequate as an explanatory lens" (ibid., p. 168). Furthermore she alludes that "chain is not the best metaphor" (ibid., p. 170) because of the "nested quality" (ibid.) and so she illustrates this continuous process using the image of the Russian nested dolls (ibid., p. 171; also Presmeg, 2001, p. 7). She describes this continuous process of semiosis as a three-step linear process in which a triad is simultaneously the object of a further triad. Her "nested model of semiotic chaining" is described very elaborately (Presmeg, 2006 p. 169 ff.; Fig. 5).

However, the processes I have reconstructed are not in all cases linear. According to my analyses, there are interpretants that serve as representamen in the following triad, and groups of sign triads that serve as representamen in a new sign triad. Furthermore, there are sign triads that are connected with one another because they correspond with the same representamen. This is depicted in Fig. 6 abstractly, and corresponds with the example in Fig. 7 which is a detail of the semiotic process card in Section 7.2 (see Figs. 6 and 7 and the semiotic process card).

Due to the non-linear alignment of the process in my example, I rejected the term "chaining" and chose instead to use the term "complex semiotic process". In my opinion, this term reflects the development of the interpretation process accurately, although these processes are partially linear. Hoffmann (2006) describes my use of the term "complex semiotic process" as a further development of the "nested chaining model", devised by Norma Presmeg with reference to the construction of meaning in educational settings (Presmeg, 2006, p. 175).

The semiotic equivalent of the negotiation of meaning which takes place during interaction, and which gives a first impression of the communication that takes place leading up to the creation of "taken as shared meaning", is the complex semiotic process. It characterizes a process leading to the construction of meaning, which can be roughly described as going from the "direct" via the "dynamic" and ultimately to the "final interpretant" (see Nöth, 2000, p. 64). Hoffmann (2002) notes that after the conduction of a complex semiotic process, the interpretant can become "the general meaning of a sign" or a "change of habit" (p. 62),[5] just as, at the end of a process of construction of meaning, the result can lead to the creation of a new frame.

5.4 Diagrams as particular signs

Many publications point to the special role that Peirce's definition of diagrams plays in educational settings in general and in mathematics education in particular (Dörfler, 2006;

[5] Hoffmann refers also to Peirce's term "habit change" in CP 5.476.

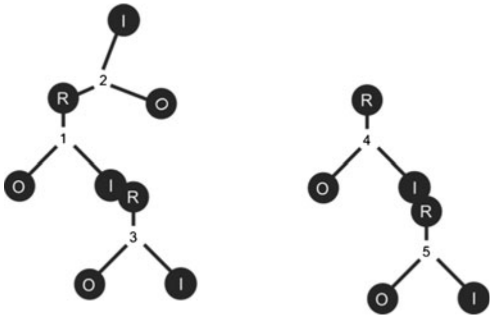


Fig. 6 The complex semiotic process

Hoffmann, 2003; Kadunz, 2006; Krummheuer, 2008). Dörfler (2006, p. 200 ff.) especially elaborates on viewing the learning of mathematics as participation in diagrammatic practices (see also Dörfler, 2004). His position is characterized by the idea that the objects of mathematical activities are "diagrams", which are devised, created, and described, and thus lead to the formation of mathematical concepts.

Using numerous examples, Dörfler (2006, 2004) demonstrates how abstract objects consistently form the central point of interest in mathematics education and how activities using illustrations and other representations are only to be viewed as support resources. This

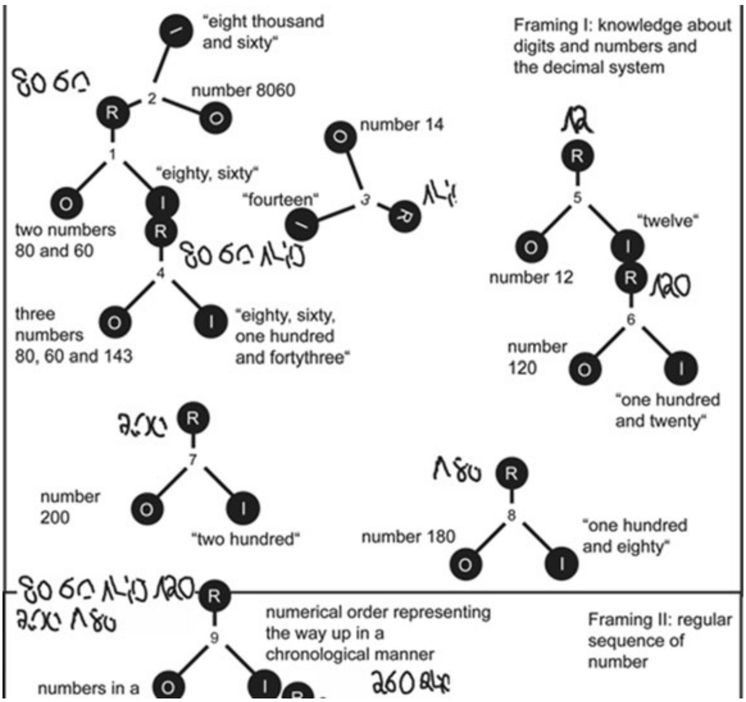


Fig. 7 Detail from a semiotic process card in Section 7.2

view of mathematics, which is based on the "immaterial nature of mathematical objects" (Saenz-Ludlow, 2006, p. 183), is also dominant in semiotic articles on the didactics of mathematics (see Steinbring, 2006; Hoffmann, 2006). Based on this perspective of mathematics as "the science of abstract objects", Dörfler advocates a shift of focus towards mathematical activities as an activity with material, perceptive, and manipulative inscriptions (Dörfler, 2006, p. 203; see also 2004). In summary, I would like to illustrate some important aspects of the concept of diagrams referred to by Dörfler (2006, p. 210 ff., 2004, p. 5 ff.):

- Diagrams are a kind of inscription, not isolated, single inscriptions, but rather part of a system of very structured depictions that provide the means with which to construct and read the inscriptions.
- A diagram is determined by conventions and a type of legend is needed to compile and read it. This legend does not necessarily have to be explicit, but rather can be learned through exposure to diagrams (Dörfler, 2006). So diagrams are "imbedded in a complex context and discourse which better is viewed as social practice" (Dörfler, 2004, p. 8).
- "Diagrams are extra-linguistic signs. One cannot speak the diagram, but one can speak about the diagrams" (Dörfler, 2004, p. 8; see also Dörfler, 2006, p. 210).
- "Intensive and extensive experience with manipulating diagrams (…) supports and occasions the creative and inventive usage of diagrams" (Dörfler, 2004, p. 7).
- Diagrams are the underlying objects of research of diagrammatical thinking, making mathematics a perceptive empirical and not only mental practice (Dörfler, 2006, p. 211).
- Diagrams can be analyzed and manipulated "irrespective of what their referential meaning may be. The objects of diagrammatical reasoning are the inscriptions themselves" (Dörfler, 2004, p. 7; see also Dörfler, 2006, p. 211).
- The possibility of observing, describing, and communicating about diagrammatic thinking in the form of operations with inscriptions makes the materiality of mathematical activities explicit (Dörfler, 2006).

The manipulation of formulae, numerals, and figures and the "intimate experience with several diagrammatical inscriptions, their structure and operations" (Dörfler, 2006, p. 213) are mentioned as examples of the basic form of the utilization of diagrams. Dörfler (2006) views the next stage as being mainly concerned with experimentation with diagrams and the exploration of their specific characteristics (p. 213). The theory then proceeds with the relationship between different diagrams, which pupils at primary level encounter as different forms of representation (Repräsentationsformen). At this point, Dörfler (2006) once again criticizes the fact that these variations of forms of representation are only used for learning "abstract objects" (p. 214). He points out the particular role of pupils' own creation and design of diagrams. All of these diagrammatic activities are embedded in the social practices of a group. Such social practices are constitutive of the meaning and significance of the diagrams. Dörfler (2004) emphasizes that the meaning is not the result of a preexisting "referential meaning" (p. 7). Thus, through operating with the diagram within social processes, its meaning emerges and can also be changed.

6 Semiotic process cards

Interaction analyses were carried out based on written transcriptions of the "Math-Chat" sessions (see Section 7). These analyses allow a detailed description of the sessions, which in turn facilitate a combined or summarized interpretation (see Krummheuer & Naujok, 1999). These interpretations were then used to describe the complex semiotic process. The results of these descriptions are demonstrated/displayed below as “Semiotic Process Cards” (henceforth SPC). In the SPC the elements described in chapter 5 are all accounted for: Peirce's triadic sign relation, embedded in an underlying “frame”, and its development as part of a complex semiotic process. Below, I first illustrate the format of the SPC; the specific SPC used here for demonstration will be analyzed in detail in Section 8.

The SPC should be read from above to below and in general from left to right. In order to make orientation easier, the triads are chronologically numbered. The letters “R” for representamen, “I” for interpretant, and “O” for object indicate which part of the triad is being referred to. Words or text and images will be used to display and demonstrate the three correlates. In some cases, the interpretant of a triad will be supplemented by further aspects and thus become a new representamen (e.g., Fig. 7, triad 3 and 5).

The frame, which is recreated through the interaction analysis, is referred to in the semiotic analysis and is indicated in the SPC. After the compilation of several SPC, the various reconstructed frames were able to be classified as “mathematical”, “argumentative”, “formal”, and “social” frames (see Schreiber, 2010).

The complex semiotic process is represented by the configuration of the triads. As Fig. 7 shows, the process can progress very differently. Where the progress is linear, the subsequent triad with the correlate “representamen” is positioned at the correlate “interpretant” of the preceding triad (see Fig. 6, triads 5 and 6). Where two parts of the process relate to the same representamen, I assign the representamen in question to two triads (see Fig. 6, triads 1 and 2). If the representamen of a triad corresponds with the entire previous process, from an accumulation of sub-processes, then the correlate representamen is placed on a line of the box which underlies the hitherto existing process (e.g., Fig. 7, triad 9).

7 An empirical example

In the chat episode presented here, two pupils, one on either side of the chat connection, solve mathematical problems. The example is depicted as an episode by using a transcription and a summarized version of its interpretation. The semiotic analysis is then demonstrated using the semiotic process cards.[6] During the episode two groups, nicknamed *Sleepers* and *Flippers* respectively, work on the following mathematical problem:

> A snail sits in a well which is 3.2 m deep. It begins to climb up the wall. During the day it climbs up 80 cm. During the night it slides down 20 cm. How many days does it take for the snail to climb to the top of the well? (See transcription 1a).

This type of mathematical problem is well-known and describes a classical problem, which is found in Adam Ries'[7] times (Krauthausen & Scherer, 2001, p. 107), but which also appears in modern mathematics textbooks. These types of exercises are also used in teacher training courses on the topic of “improving problem solving though drawings" (Kelly, 1999). For me, the presentability of the problem in picture, sketch, and table form, etc. was the most important reason for the choice.

[6] This and other contrasting examples are described in detail in Schreiber (2010).

[7] Adam Ries (*1492 – +1559) was a German mathematician.

7.1 Interpretation of the episode

In the interpretation presented here, all the information in brackets refers to parts a and b of the transcription. The utterances have been abbreviated as follows: (S1, 3) for example, stands for sleepers 1, utterance 3.

We were able to divide the communication that took place in the chat into three interconnected processes. The process of creation of the inscription on the Sleepers' side, the process of chat communication on the basis of this inscription and the process of interpretation of the inscription on the Flippers' side. The inscription is constructed solely by the Sleepers. The construction of the individual parts of the inscription by the Sleepers takes place without further consultation, hand in hand (S1 and S2, 14–34): both pupils understand the meaning and composition of the jointly created inscription. Although the meaning of the sequence of numbers is not easily accessible to an outside observer, it is possible for both pupils continually to predict, precalculate, suggest, and adjust the next part of the inscription on this page. The result (in days) which cannot be directly read, can be gathered by both from the inscription (S1 and S2, 35–44).

The Flippers' interpretation is characterized by the process of reception of the inscription created and "published" by the Sleepers. The Flippers comprehend and reconstruct the writing process. However, they cannot reliably identify the individual numbers from the arrangement of numerals. From their point of view, the numbers stand adjacent to one another without being connected and their relation to the problem is not clear. This changes after the number 180 is written by the Sleepers. From this point on, Flippers 1 presumes that a legitimate sequence of numbers is being presented and comments on parts of the inscription and their meaning (F1, 7; F1, 8). Flippers 1 can reassess this assumption through the algorithmic continuation of the numeric sequence, and this seems to affirm and validate his assumption (F1, 8; F1, 10; F1, 11). The result of the exercise is thereby readable, at least for him. His partner approves this outcome and suggests giving the Sleepers a positive response (F2, 12).

7.2 Semiotic analysis from the Flippers' perspective

This type of summarized analysis serves as a basis for the reconstruction of the complex semiotic process. For this reason, I first developed the so-called "semiotic learning cards" (Semiotische Lernkarten; Schreiber, 2005a, b). During the advanced process of analysis, these were renamed as "semiotic process cards" (Schreiber, 2010), as the processes described here are predominantly semiotic and not necessarily learning processes.

The information in brackets alludes to parts a and b of the transcription. The triads correlate with the semiotic process cards (see Fig. 8), in which this analysis is presented graphically. The semiotic analysis of this scene is presented here from the perspective of the Flippers.

The representamen in triad 1 is the beginning of the inscription, and generates two different interpretants, namely "eighty, sixty" and "eight thousand and sixty" (in F1, 6). The first interpretant shows that the inscription is being read as a representamen for the object "two two-digit numbers", namely 80 and 60. The same inscription is also read as the object "one four-digit number", namely 8060. Both understandings imply a similar frame: the "frame as a decimal depiction of numbers". This frame is based on socially shared knowledge of the decimal system and the correlation between digits and numbers (frame 1). The digits "1" and "4" follow (in S2, #7), which are read as "fourteen", a two-digit number (F1, 6). When the Sleepers extend the inscription by "0" (S2, #7), the Flippers refer to the

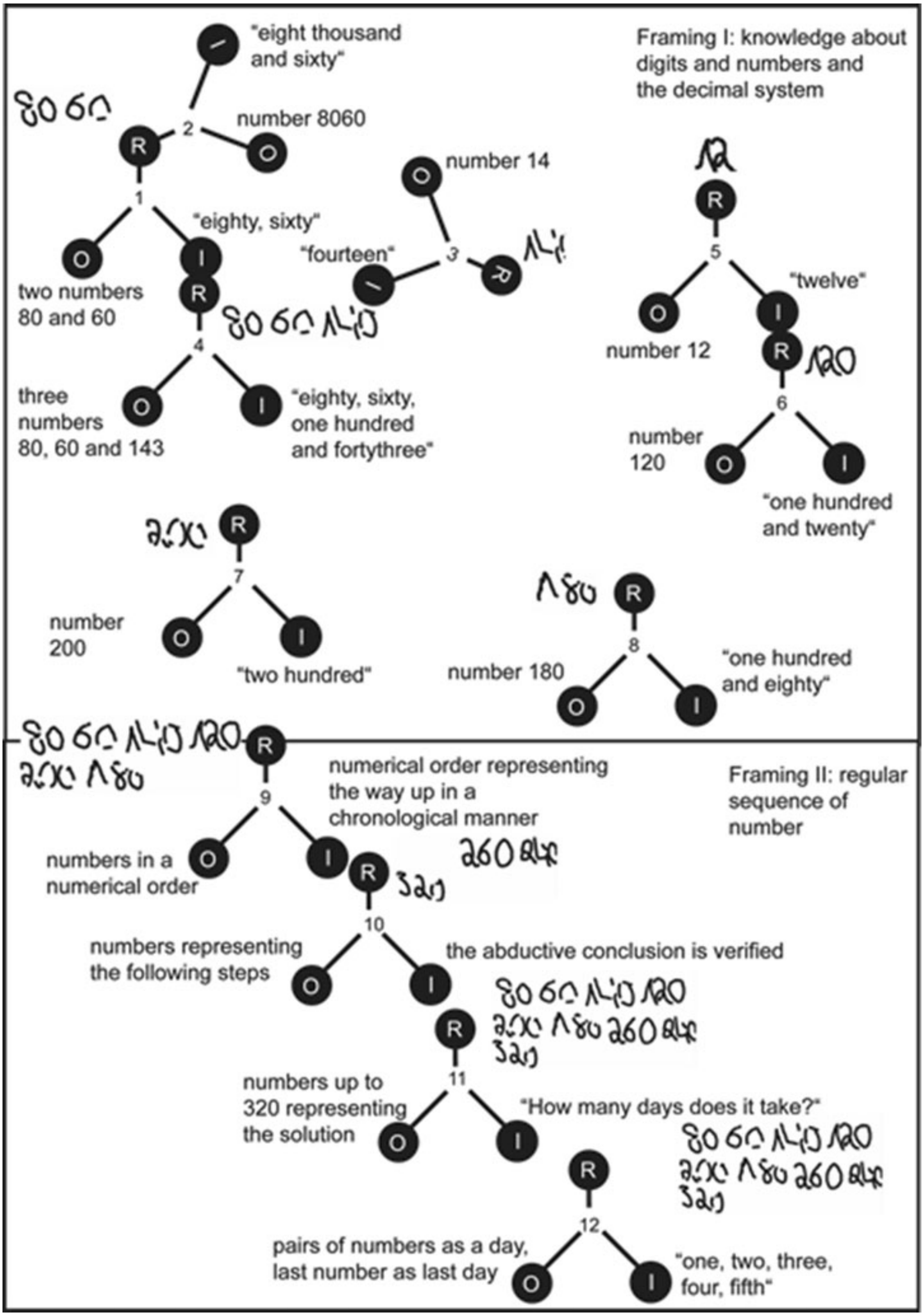


Fig. 8 Semiotic process card from the perspective of the Flippers

interpretant from the first triad "eighty, sixty" (F1, 6) again. They construe the entire inscription up to this point "80 60 140" (whiteboard excerpt at 30:50) as the representamen of the object "80, 60, and 143", which is therewith made up of three separate numbers (F1, 6). The last digit is read as "3" (F1, 6). This, however, is not discussed further. Frame I, the "frame as a decimal depiction of numbers" still forms the basis of the interpretation.

Subsequently, the Flippers refer only to that part of the inscription shown in triad 5. On the basis of frame I, this representamen elicits the interpretant "twelve" (F1, 6). When the digit "0" is added to the inscription, the representamen in triad 6 produces the interpretant "one hundred and twenty" (F1, 6), which represents the three-digit number 120. In the same way, the representamen in triads 7 and 8 produce interpretants based on frame I. In the following triad, a change of focus occurs which is crucial to the problem-solving process: the Flippers no longer relate only to individual parts of the inscription, but to the inscription as a whole (see Fig. 8 of triad 9), as it has been developed up to this point. In other words, they relate to the interpretant which has so far been generated by the inscription. One of the Flippers comes to the following abductive conclusion: he recognizes the representamen from triad 9 as a representation of a regularity in the development of a sequence of numbers. The basis of this comprehension is assumed to be the "frame as a regular sequence of numbers", frame II. The pupil has grasped the concept of consecutive numbers as a sequence defined by its regularity, and the idea that this sequence of numbers reflects the stages of the mathematical problem.

Further examples were chosen specifically by me due to their partially analogue development or, in other cases, precisely due to their oppositional development. After they were analyzed, they were presented as SPCs and the complex semiotic processes were compared. The abductive conclusions the pupils reached and the associated adjustment of the frame caught my particular interest. These conclusions and the adjustments in the frame proved to be central to the utilization of inscriptions. In some episodes, these could then be used diagrammatically after the adjustment. The findings, which I present in brief in the following section, are the result of the analysis and comparison of numerous examples.

8 Findings

In order to present my findings effectively, I refer to the outline, use and development of inscriptions, the use of diagrams as a particular sign and to the complex semiotic processes within the developing frames. It must be noted at this point that the inscriptions used by the pupils involved in the research described here were designed and used as mutual inscriptions. Where the inscriptions are used collectively, one can refer to them as being mutual inscriptions, even when these are generated from only one side of the chat setting, as in the example given. In a continuation of this example, it will be demonstrated how this inscription can be used mutually, and reconstructed as a mutual inscription and applied productively in an analogue situation.[8]

In this example, an abductive conclusion is reached in order for the inscription to be created. This conclusion is a key factor in the understanding, and thus utilization, of the inscription. The abductive conclusion is the prerequisite for the conversion of one group's inscription into the mutual inscription of both participating groups. As Meira (2002, p. 95) describes, the inscription is applied in a minimalist fashion; only what is absolutely necessary is contained therein.

[8] See Schreiber 2006 and chapter 5.3 in Schreiber 2010.

Diagrams are developed at this point, which facilitate a productive problem-solving process. In reference to Dörfler (2006, p. 210 ff.), I wish to list some characteristics of diagrams, which can be found in the examples:

- The inscriptions described here are not single and isolated, but form part of a system of diagrams.
- There is a type of "legend" (ibid., p. 210) which is not taken as a given, but which was developed out of experience through exposure to and use of the diagrams.
- The diagrams are discussed and spoken about orally, but they appear in written form, in this case as inscriptions on a computer screen.
- As the diagrams in this form are usually not familiar in educational settings (example 1), it is assumed that new diagrams are constructed, which are then formalized.
- The inscription of the participants on one side of the chat setting becomes an "object of research" for the participants on the other side (ibid., p. 211).

The various semiotic processes are clarified in the semiotic process cards. At the same time, the individual triads represent the analysis on the smallest possible micro level. During the process, a sign, a representamen is analyzed in detail in order to ascertain which interpretants it generates. The interpretant, which Peirce referred to as an inner sign, can only be determined by the utterances that follow the representamen. During the analysis, it can be determined which object the relation representamen-interpretant stands for, in other words, what the observer believes the creator of the representamen to have demonstrated. In this context, all three parts of Peirce's sign relation in their entirety are considered to represent the sign. What is crucial is the perspective of the person or persons who perceive the representamen as a sign, and thus set the process of interpretation in motion.

It is therefore possible at this point to illustrate the progression of the complex semiotic process through the configuration of the triads. These are arranged singly or in strands and are examples of either linear processes, as with the "chaining" process described above, or non-linear processes, as in a complex semiotic process. The triads correspond partly with the same representamen, yet they create different interpretants, either despite or because of the fact that they have the same frame. In this way, the interpretant of a triad can become the representamen of a further triad. In my opinion, only the interpretant generated and the oral or written statement resulting from it can serve as representamen of further triads. In some cases, it is not a single interpretant but a summation of tried and tested interpretants that make up the representamen of the following triad. These are thus the catalyst for the continuation of the process.

The respective frame that has been activated, which I also based graphically in the SPC process, considerably determines these processes. The reconstruction of the frame is the result of detailed interaction analyses, and it is referred to in the semiotic analysis that follows each interaction analysis. These frames sometimes reappear at a later point in time in the processes, while other frames develop further. This development takes place partly as a result of new representamen, which are perceived by the interpreter as such. However, in some cases the changes in the frame are the result of a new interpretation of prior knowledge.

What is specific to the chat in this case is that the time and effort needed for the coordination of the process is much higher than in face-to-face situations in which a group can clarify social processes orally and through gestures. The time invested in the *argumentative* situations is also notably higher in the chat settings. What is remarkable is that in those situations where diagrams were created and used, this effort is largely absent, as precisely these *arguments* are evidently already inherent to the user through the utilization of the diagrams.

It is an abductive conclusion, which leads to the change in the frame, which forms the basis of the process in SPC 1. It is precisely such changes in the frame, which caught my special attention. They do not occur very often, yet they constitute very clear steps in the development of the problem-solving process. It was possible to recreate one of the abductive conclusions here. As a result of this reconstruction, it became clear how the suggestions put forward for those diagrams that ultimately lead to the solution, then surpassed the solution of the first examples to become mutually used diagrams. This seems to me to be the step towards a "final interpretant" (CP 4.536), which can then serve as a frame in subsequent examples.

This example shows how the successful development of the pupils' own inscriptions in a written or graphical based problem-solving process can progress. What takes place in this example is what Hoffmann (2002) refers to as the "change of habit" ("Veränderung einer Gewohnheit"; p. 62; CP 5.476) in reference to the development of an interpretant. From an interactions-theoretical point of view, this corresponds with the (preliminary) end of a "negotiation of meaning", and as such leads to a new frame.

Working with the inscriptions, which were created and developed by the pupils, can be very productive for problem-solving processes, as shown above. The inscriptions in question were especially effective when they did not constitute solutions or parts of solutions, but rather significant elements ultimately leading to the solution. The use of the inscriptions was shown to be especially productive when they were mutually created by the participants (or a mutual generation was at least possible). The observation of the starting point of the process was especially helpful in order to facilitate its further formation and utilization.

In this way, the use of inscriptions in collective problem-solving processes should also be made possible through working together on these inscriptions in such a way that all participants can observe the construction of these inscriptions and can contribute. The requirements for the creation of mutual inscriptions are especially advantageous under these conditions. In this way, the use of inscriptions in collective problem-solving processes is made possible.

Inscriptions, once they have been constructed and used productively, can then be retrieved anew in subsequent problem-solving processes. At the same time, a gradual development, in terms of an enhancement or formalization, takes place. The mutually created and successfully implemented inscriptions can then be activated as (part of) a new frame.

In this way, the inscriptions can become self-created or jointly created diagrams, whose value for the problem-solving process and whose mathematical significance is extremely high. At this point, it is possible to be very productive mathematically. Working with diagrams as a central mathematical activity/operation can lead to the creation of what Dörfler (2006) calls "successful diagrammatical thinking" (p. 216). The necessary "intimate experience" (Dörfler, 2004, p. 8) is given through the involvement in its generation, the observation of its generation, its amendment etc. It is precisely this type of diagram, embedded in social practice, which makes the problem-solving process fruitful for learning. The empirical evidence presented in this paper corresponds greatly with Dörfler's theses on the importance of diagrams for academic mathematics and mathematical learning processes.

These conclusions pertaining to the inscriptions generated by the pupils and the diagrams that are subsequently generated should be taken into consideration in problem-solving situations initiated by teachers as they present the following possibilities:

Pupils

- work together in such a manner as allows all of them to participate in the compilation of a graphical or written representation;

- put the problem-solving process into writing without being bound by formality while doing so;
- make the written form a central topic of the process and of discussion;
- have caused to try out their own customary methods anew;
- are given the opportunity to optimize the inscriptions and thus to develop diagrams in the best sense;
- can become aware of the importance of the self-generated diagrams.

The semiotic process cards presented above were developed as a possibility to describe and present the analysis of collective problem-solving processes in their individual elements on a micro level and also to display their progress graphically. This form of presentation makes a comparison with other processes accessible. In my opinion, processes in which representamen are available in written form are especially appropriate for presentation, as an analysis based on Peirce's semiotics is especially effective relative to such processes.

The changes in relation to the frames, which are triggered by new representamen, other focusing or abductive conclusions, provide very interesting insights into the progress of the process. The pupils' use of the diagrams they generated themselves and the activation of the same frame as a frame for analogue problems, demonstrate the learning curve of the participants. In the process, frames can be classified and it is thus possible to compare processes while they are taking place.

Appendix

Transcription rules

1. column and 7. column

- line numbers and time

2. column and 6. column

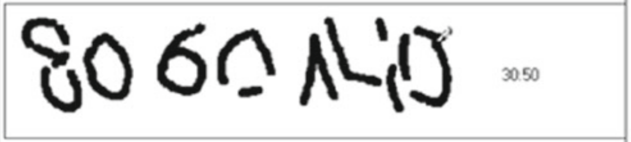
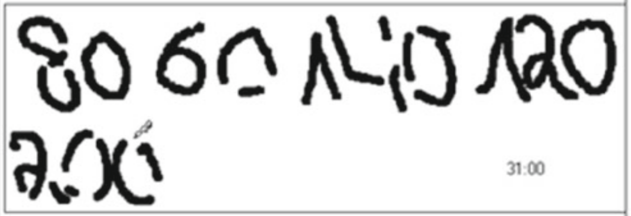
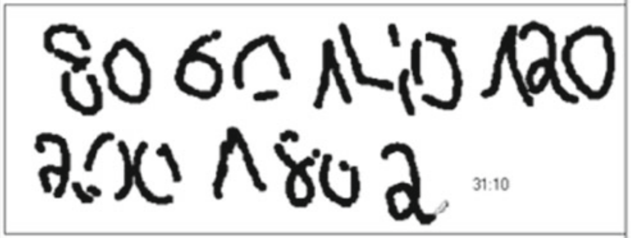
- shortnames of interacting persons on the left hand side (Times New Roman 12 pt bold).
- oral utterances on the right hand side (Times New Roman 12 pt); incomprehensible utterances are marked as (incomprehensible).
- paraverbal information, (special characters see below), for example *emphasizing*, *whispering*, etc. (Times New Roman 12 pt italics)
- # refers to actions on the computer

3. column and 5. column

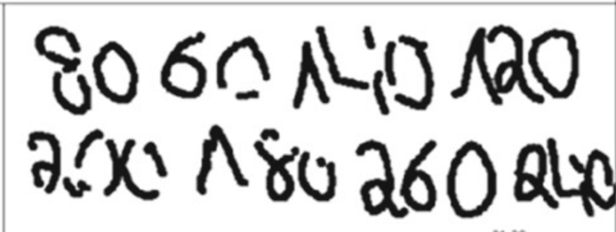
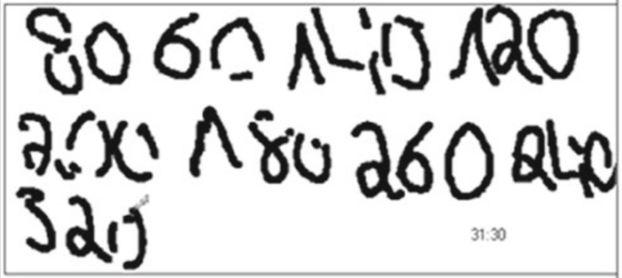
- actions marked with # are actions on the computer of each chat participant.

4. column

- part of the screenshot with time information (here every 10 seconds)

line/ time	oral utterances Sleepers	activities Sleepers	whiteboard	activities Flippers	oral utterances Flippers	time/ line
30:40						**30:40**
14	**S2:** this is [#5 s i x t e e n\ #5] 60	[#5 **30:41** writes 60 onto the white-board]			**F1:** 80 60\ 8060 yes\ (..) 14 80 60 143	6
15	**S2:** <plus 80 is					
16	**S1:** < 140\					
17	**S2:** [#6 140 #6]	[#6 **30:46** writes 140]			(..) yes\ 12 120\ (...)	**30:50**
30:50						
18	**S1:** (..) em\ (..)					
19	**S1:** <[#7 120\ (.)	[#7 **30:52** writes 120]				
20	**S2:** < 120\					
21	**S1:** #7] Plus 80\ is [#8 200\ #8] minus 20\	[#8 **30:58** writes 200]			200\ (5sec.)	**31:00**
31:00 22	**S2:** is [#9 180\ #9] plus 80\ is	[#9 **31:04** writes 180]			**F1:** 180\ (...) I see\ 80 is minus, so it is 60\ 60 plus 80\	**31:10** 7
31:10	[#10	[#10 **31:10** writes 260]				
23	**S2:** < 260\ #10]					
24	**S1:** < 60\					
25	**S2:** >[#11 240\	[#11 **31:15** writes 240]				
26	**S1:** > 240\					
27	**S2:** <#11] Plus 80					
	S1: < 80					

Transcription: part a

line/ time	oral utterances Sleepers	activities Sleepers	whiteboard	activities Flippers	oral utterances Flippers	time/ line
31:20 29 30 31 32 33 34	 **S1**: > is two- **S2**: > is\ **S1**: <hundred- **S2**: <hundred and **S2**: twenty **noo** **threehundred-** twenty\ 320\ **S1**: *whispering* [#12 320\ #12]	[#12 **31:26** writes 320]	 31:20		is 140\ (.) minus 20 120\ **F1**: <120 plus 80 **F2**: < mmh\ **F1**: is 200\	**31:20** 8 9 10
31:30 35 36 37 38	 **S1**: **yes** to say **S1**: < 1 2 3 4 **S2**: < 1 2 3 4 **S1**: **5** days\		 31:30		**F1**: plus (.) minus 20 is 80 260 240 320\ how many/	**31:30** 11
39 **31:40** 40 41 42	**S2**: **Noo** 4, 1 2 3 4 noo let us calcu- late \ **noo** wait **S2**: < 1 2 3 4 5 **S1**: < 1 2 3 4 5 **S1**: yes\ it is 5 days\		Whiteboard does not change		how many days does it take to crawl up / 1 2 3 4 fifth\ (7 sec.)	**31:40**
43 **31:50** 44	**S2**: 5 days or/ 5, 5 **yes 5** **S1**: 5 d a y s \				**F2**: should we write right/	**31:50** 12

Transcription: part b

Special characters:

,	short break in an utterance
(.)	break (1 s)
(..)	break (2 s)
(…)	break (3 s)
(4 s)	duration of a break longer than 3 s
/ - \	rising, even, falling pitch
yes	**bold**: accentuated word
s i x t e e n	s p a c e d: spoken slowly

(“<”) and (“>”) two participants are talking both at the same time, for example:
8 S2: < plus 80 is 140/
9 S1: < 140\ ok

References

Bateson, G. (1955). A theory of play and fantasy. In Bateson, G. (1972), *Steps to an ecology of mind* (pp. 177–193). New York: Ballantine.

Blumer, H. (1969). *Symbolic interactionism*. Prentice-Hall, NJ: Englewood Cliffs.

Bruner, J. (1996). *The culture of education*. Cambridge, MA: Harvard University Press.

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Hillsdale, NJ: Lawrence Erlbaum.

Donaldson, M. (1978). *Children's minds*. London: Fontana/Croom Helm.

Dörfler, W. (2004). Mathematical reasoning: mental activity or practice with diagrams. In M. Niss (Ed.), *Proceedings of the Tenth International Congress on Mathematical Education*. Roskilde: IMFUFA, Roskilde University.

Dörfler, W. (2006). Diagramme und mathematikunterricht. [Diagrams and mathematics instruction]. *Journal für Mathematik-Didaktik, 27*(3/4), 200–219.

Dürscheid, C. (2003). Medienkommunikation im Kontinuum von Mündlichkeit und Schriftlichkeit. Theoretische und empirische Probleme. [Media communication in a continuum of vocality and writing. Theoretical and empirical problems]. *Zeitschrift für Angewandte Linguistik, (38)* (pp. 37-56), Koblenz: Peter Lang Verlag.

Eco, U. (1976). *A theory of semiotics*. Bloomington, Indiana: Indiana University Press.

Eco, U. (1979). *The role of the reader: Explorations in the semiotics of texts (advances in semiotics)*. Indiana: Indiana University Press.

Fetzer, M. (2007). *Interaktion am Werk. Eine Interaktionstheorie fachlichen Lernens, entwickelt am Beispiel von Schreibanlässen im Mathematikunterricht der Grundschule. [Work-based interaction. A theory of interaction of subject learning, developed using writing impulses in primary mathematical education].* Bad Heilbrunn: Klinkhardt

Goffman, E. (1974). *Frame analysis. An essay on the organisation of experience*. Cambridge, MA: Harvard University Press.

Gravemeijer, K. (2002). Preamble: from models to modeling. In K. Gravemeijer et al. (Eds.), *Symbolizing, modelling and tool use in mathematics education* (pp. 7–22). Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K., Cobb, P., Bowers, J. S., & Whitenack, J. (2000). Symbolizing, modelling and instructional design. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms* (pp. 225–273). London: Lawrence Erlbaum.

Hoffmann, M. (1996). *Eine semiotische Modellierung von Lernprozessen. [A semiotic modeling of learning processes]*. Occasional Paper 160, retrieved from http://www.unibi.de/idm/serv/dokubib/occ160.pdf

Hoffmann, M. (2002). *Erkenntnisentwicklung—Ein semiotisch pragmatischer Ansatz. [Development of cognition—a semiotic pragmatic approach]*. Habilitation. Philosophische Fakultät Technische Universität Dresden.

Hoffmann, M. (Ed.). (2003). *Mathematik verstehen. Semiotische Perspektiven. [Understanding mathematics. Semiotic perspectives]*. Hildesheim: Franzbecker.

Hoffmann, M. (2006). What is a “semiotic perspective”, and what could it be? Some comments on the contributions to this special issue. *Educational Studies in Mathematics, 61*(1–2), 279–291.

Kadunz, G. (2006). Experiments with diagrams—a semiotic approach. *Zentralblatt für Didaktik der Mathematik, 38*(6), 445–455.

Kelly, J. (1999). Improving problem solving through drawings. *Teaching Children Mathematics, (6)* 1, 48-51.

Krauthausen, G., & Scherer, P. (2001). *Einführung in die Mathematikdidaktik. [Introduction to the didactics of mathematics]*. Heidelberg: Spektrum Akademischer Verlag.

Krummheuer, G. (2000a). Interpretative classroom research in primary mathematics education. Some preliminary remarks. *Zentralblatt für Didaktik der Mathematik, 32*(5), 124–125.

Krummheuer, G. (2000b). Mathematics learning in narrative classroom cultures: Studies of argumentation in primary mathematics education. *For the Learning of Mathematics, 20*, 22–32.

Krummheuer, G. (2008). Inskription, Narration und diagrammatisch basierte Argumentation. Narrative Rationalisierungspraxen im Mathematikunterricht der Grundschule. [Inscription, narration and diagramatic argumentation. Narrative rationalization practices in mathematical education in primary schools]. In H. Jungwirth & G. Krummheuer (Eds.), *Der Blick nach innen: Aspekte der alltäglichen Lebenswelt Mathematikunterricht (2)* (pp. 7–37). Münster: Waxmann.

Krummheuer, G., & Fetzer, M. (2005). *Der Alltag im Mathematikunterricht. Beobachten, Verstehen, Gestalten. [Mathematics education in everyday life. Observing, understanding, constructing]*. Heidelberg: Spektrum Akademischer Verlag.

Krummheuer, G., & Naujok, N. (1999). *Grundlagen und Beispiele Interpretativer Unterrichtsforschung. [Basic principles and examples of interpretativ classroom teaching research]*. Opladen: Leske und Budrich.

Latour, B. (1987). *Science in action: How to follow scientists and engineers through society*. Cambridge, MA: Harvard Press.

Latour, B. (1990). Drawing things together. In M. Lynch & S. Woolgar (Eds.), *Representation in scientific practice* (pp. 19–68). Cambridge, MA: MIT Press.

Latour, B., & Woolgar, S. (1986). *Laboratory life. The construction of scientific facts* (2nd ed.). Princeton: Princeton University Press.

Lehrer, R., Schauble, L., & Carpenter, S. (2000). The interrelated development of inscriptions and conceptual understanding. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms* (pp. 325–360). London: Lawrence Erlbaum.

Meira, L. (1995). The microevolution of mathematical representations in children´s activities. *Cognition and Instruction, 13*(2), 269–313.

Meira, L. (2002). Mathematical representations as systems of notations-in-use. In K. Gravemeijer et al. (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 87–103). Dordrecht: Kluwer Academic Publishers.

Morgan, C. (1998). *Writing mathematically—the discourse of investigation*. London: Falmer Press.

Naujok, N., Brandt, B., & Krummheuer, G. (2004). Interaktion im Unterricht. [Interaction in classroom education]. In W. Helsper & J. Böhme (Eds.), *Handbuch der Schulforschung* (pp. 753–773). Wiesbaden: Verlag für Sozialwissenschaften.

Nöth, W. (2000). *Handbuch der Semiotik. 2—vollständig neu bearbeitete und erweiterte Auflage. [Handbook of semiotics. Second edition]*. Stuttgart, Weimar: Verlag J.B. Metzler.

Otte, M. F. (2011). Evolution, learning, and semiotics from a Peircean point of view. *Educational Studies in Mathematics, 77*, 313–329.

Peirce, C. S. (1931-1935). *Collected papers of Charles Sanders Peirce. Volumes I-VI*, (ed. by C. Hartshorne & P. Weiss 1931-1935; quotations, as established, according to volume and paragraph). Cambridge, MA: Harvard University Press.

Pimm, D. (1987). *Speaking mathematically. Communication in mathematics classrooms*. London, New York: Routledge.

Presmeg, N. (2001). *Progressive mathematizing using semiotic chaining.* The 25th Annual Conference of the International Group for the Psychology of Mathematics Education, Discussion Group 3, Semiotics in Mathematics Education. Retrieved from http://www.math.uncc.edu/~sae/dg3/norma-PME25DG.pdf

Presmeg, N. (2006). Semiotics and the "connections" standard: Significance of semiotics for teachers of mathematics. *Educational Studies in Mathematics, 61*, 163–182.

Reinhard, C. (2008). Wiki-basierte Lernumgebung zum kooperativen Lernen mit Neuen Medien im Mathematikunterricht der Primarstufe—WiLM@. [Wiki-based learning environment for cooperative learning with new media in the primary mathematics classroom—WiLM@] In *Beiträge zum Mathematikunterricht* (pp. 657–661). Hildesheim: Franzbecker.

Roth, W.-M., & McGinn, M. (1998). Inscriptions: Toward a theory of representing as social practice. *Review of Educational Research, 68*(1), 35–59.

Saenz-Ludlow, A. (2006). Classroom interpreting games with an illustration. *Educational Studies in Mathematics, 61*, 183–218.

Schreiber, C. (2004). The interactive development of mathematical inscriptions—a semiotic perspective on pupils' externalisation in an internet chat about mathematical problems. *Zentralblatt für Didaktik der Mathematik, 36*(6), 185–195.

Schreiber, C. (2005a). *Semiotic processes in a mathematical internet-chat.* In ERME: Proceedings 2005 (CERME 4 in Sant Feliu de Guixols). http://ermeweb.free.fr/CERME4/CERME4_WG8.pdf

Schreiber, C. (2005b). Semiotische Lernkarten. [Semiotic learning cards]. In *Beiträge zum Mathematikunterricht* (pp. 525-528). Hildesheim: Franzbecker.

Schreiber, C. (2006). Die Peirce'sche zeichentriade zur analyse mathematischer chat-kommunikation. [Using the Peircean sign-triad to analyse mathematical chat communication]. *Journal für Mathematikdidaktik, 27* (3/4), 240–267. Wiesbaden: Vieweg+Teuber Verlag.

Schreiber, C. (2010). *Semiotische Prozess-Karten—chatbasierte Inskriptionen in mathematischen Problemlöseprozessen. [Semiotic Process Cards—chatbased inscriptions in mathematical problem-solving processes]* Dissertation. Münster: Waxmann

Seeger, F. (2011). On meaning making in mathematics education: Social, emotional, semiotic. *Educational Studies in Mathematics, 77*, 207–226.

Sherin, B. (2000). How students invent representations of motion—a genetic account. *Journal of Mathematical Behavior, 19*(4), 399–441.

Steinbring, H. (2006). What makes a sing a mathematical sign?—an epistemological perspective on mathematical interaction. *Educational Studies in Mathematics, 61*, 133–162.

Volkert, K. (1990). Mathematics and semiotics. In W. A. Koch (Ed.), *Semiotics in the individual sciences* (pp. 122–134). Bochum: Brockmeyer.

Zellmer, S. (1979). *Pädagogische Semiotik in der Mathematik-Didaktik. [Pedagogical semiotics in the didactics of mathematics]*. Baden-Baden: Agis.